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TORONTO

AN INTERMEDIATE COURSE

OF

PRACTICAL PHYSICS

BY

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PREFACE

THIS book was originally written to assist students preparing for the Intermediate B.Sc. and First M.B. examinations of the Victoria University; but the requirements of these examinations are so nearly identical with those of other Universities, and the courses of Practical Physics given in Public, Technical, and other Schools, cover so nearly the same ground, that it is hoped it will prove useful to a wider circle of students.

The term *Intermediate* in the title of the book, has been used to indicate that its standard is higher than that of the Elementary Text Books; but it is by no means essential that students should have worked through an elementary book in order to perform the exercises described in this volume.

The book is in the main, a reprint of the papers of instructions which have been issued to the students of the Owens College during the last five years.

In order that a large class might be taught simultaneously, it was necessary that the explanations should contain sufficient detail to allow each student to do his work without individual supervision; but, at the same time, we had to guard against the danger of giving such minute instructions as would allow a student to perform an experiment and obtain results in a mechanical manner, and without a proper understanding of what he was doing.

The experiments have been performed each year by nearly a hundred students, and have been altered until experience has shown that the explanations given were sufficient, and that good results could be obtained, although the apparatus was in each case reduced to its simplest form.

The exercises have been designed in such a way that it is not necessary to use apparatus identical with that described in the book. To some extent the educational value of the book would be increased, if the instruments used differed from those mentioned in the text, so that the students might have to exercise a little ingenuity in varying the experiment to suit the apparatus supplied. Much must however depend on the available teaching power. If one teacher has to supervise a great number of students, he will probably find it

advantageous to follow the book pretty closely, and to work with apparatus not differing materially from that described.

A complete set of Apparatus, details of which will be found on p. 237 of the Appendix, may be obtained for about £13, and five or six sets will be sufficient for classes of thirty students.

The course has been arranged so that an average student may pass through it in about thirty lessons of two hours each. It is divided into sections, some of which are purely explanatory; the great majority of the remainder can each be worked through in one lesson.

Many backward students may with advantage omit some of the sections which are more advanced than the rest, and on the other hand, those who already have some elementary knowledge of Practical Mechanics, may omit some of the easier exercises in Part II.

Experience has led us to attach the greatest importance to the proper keeping of note-books by the students, and to ensure this the books should be examined from time to time. The teacher should also keep such a record of the work done, as will enable him to see at a glance how each student is progressing. The Appendix contains a description of the method we have adopted to secure this end.

We beg to thank Mr. A. Griffiths, M.Sc., and Mr. J. D. Chorlton, B.Sc., who have assisted us in reading through the proofs, Mr. W. W. H. Gee, B.Sc., to whom a few of the original exercises were due, and the Students who have worked in the Physical Laboratory of the Owens College during the past five years, to whose difficulties and questions we owe the removal of many obscurities from the pages of our manuscript.

ARTHUR SCHUSTER,
CHARLES H. LEES.

THE OWENS COLLEGE,
July 1896.

CONTENTS

PART I

PRELIMINARY

PART II

MECHANICS

PART III

HEAT

For Additional Exercises in Heat, see pp. 230–232.

PART IV

LIGHT

PART V

SOUND

PART VI

MAGNETISM

PART VII

ELECTRIC CURRENTS

PART VIII

ELECTRIC CHARGES

PART I

PRELIMINARY

INTERMEDIATE

COURSE OF PRACTICAL PHYSICS

PART I.—PRELIMINARY

SECTION I

General Instructions

THE exercises in this volume are intended to train the student's faculties of observation, and to impress upon his memory some of the more important laws of Physics, which are treated in a more abstract manner in lectures and text-books. Laboratory practice is now generally recognised as an essential part of scientific instruction, but the educational value of experimental work depends entirely on the care which is bestowed upon it, and the trouble which is taken to secure accurate results.

Read carefully through the instructions.—To perform an experiment successfully, it is essential in the first place to have a clear idea of its object. Students should therefore, before beginning an experiment, read carefully through the instructions and explanations which are given in the section devoted to it. It is only when they have realised clearly what measurements they are to make,

and how these measurements are to be treated, that they are ready to begin.

Note-book of observations and calculations.—Every observation should be written down at once, and exactly in the form in which it is taken. For this purpose a note-book should be kept, and the observations entered in it in such a way, that at any future time the meaning of each entry will be perfectly clear. It is of the greatest importance that all note-books should be kept in good order.

Arithmetical calculations.—It will eventually save time if the arithmetical work is also written down in detail as it occurs, so that each step may be checked, and errors of arithmetic more easily traced. It is necessary both in the observations and calculations, that the work should show all the figures obtained, even if the last figure is zero. Thus a length may have to be measured in centimetres to the nearest millimetre; if 31 millimetres is found, it is put down as 3·1 centimetres, and if 40 millimetres is found, it should be put down as 4·0 centimetres and not as 4 only. The two numbers 13·7 and 13·700 do not mean the same thing, when put down as the result of an experiment. The former implies that the result has been obtained to three significant figures, and that no attempt has been made to determine the fourth, while the latter implies that the result has been obtained to five figures and that the two last figures have been found to be zero.

Note-book of results.—In addition to the book of observations and calculations, each student should be provided with a note-book in which the apparatus used in any particular exercise, and the theory of the experiment, should be briefly but clearly described and illustrated by diagrams, and the result of the experiment stated.

Students will find that the labour spent on this part of the work will be amply repaid. Not only will they more easily remember what they have done, but a future revision of the work will be an easy task, if their note-books are kept in good order. It is convenient if every alternate page of this book is ruled in squares.

Avoid large errors.—In their endeavour to secure good results, beginners often concentrate too much of their attention on some detail of the work, neglecting to bestow sufficient care on what to them seems easy. Thus in reading a thermometer and estimating tenths of a degree, it often happens that the whole degrees are read off carelessly and incorrectly, or in measuring a length attention is paid to the millimetre, and mistakes are made in the number of centimetres. Such errors may with a little care be avoided.

Subdivision by eye estimate.—In making a measurement we must often obtain the last figure by "estimation." Thus if a length is measured on a scale divided into millimetres, the scale shows directly between which two millimetres the length must lie; but in most cases that is not sufficient. Thus in Fig. 1 it is seen at once that the

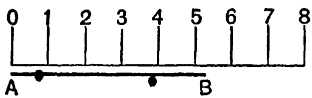


FIG. 1.

length AB is larger than 5 but smaller than 6, divisions of the scale which is drawn above it. Everyone will also see at once, that the end B lies nearer to 5 than to 6 on the scale, but some may be doubtful whether AB overlaps 5 by more or less than a quarter of a division. After a

little practice, that doubt disappears, and students will be able to estimate almost with certainty to a tenth of a division, and put down 5·3 as the required length.

Parallax.—A source of error which is liable to render many physical measurements inaccurate, may here be noticed. Supposing the length of a vertical rod is to be measured on a vertical scale with which it cannot be placed

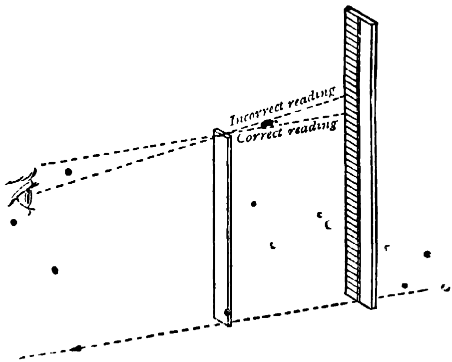


FIG. 2.

in contact. Fig. 2 will show at once, that unless the eye is so placed that the line of vision is horizontal, an error in the measurement will occur. The word "parallax" means "angle," and the word has come to be used specially for the angle formed between two lines of sight such as those indicated by the dotted lines in the figure. As the error committed in reading the scale, depends on that angle, the error is said to be due to "parallax." To avoid it, we must have some means of causing the line of sight to be horizontal—*i.e.* perpendicular to the scale. A useful device often employed in these exercises is illustrated in

Fig. 3. The scale is etched on the front surface of a plate of glass, the back of which is silvered so as to act as a mirror. It will be shown in Sec. XXII. that a line joining an object and its image in a plane mirror, cuts the latter necessarily at right angles. Hence if the eye is so placed that the end of the rod to be measured and its image just cover each other, the right position for the eye

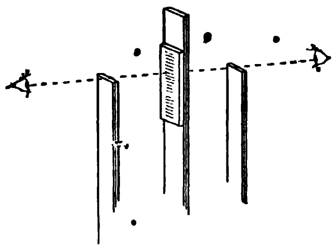


FIG. 3.

has been found. When the scale can be brought near to the object to be measured, it is sometimes found more convenient to adjust the position of the eye until the image of the eye (*i.e.* the image of the centre of the pupil) covers the point of the object to be measured. In this case also, the line of vision will be at right angles to the scale.

Accidental and systematic errors.—If an experiment is repeated, the result obtained is not always the same, since even when large errors have been successfully avoided, small differences called *accidental errors* still occur owing to the imperfection of our senses or apparatus, or to some disturbing influence which has produced some unintentional change in the apparatus or conditions. Hence it is generally advisable to repeat all observations,

the arithmetical mean of the results obtained in different experiments being probably more correct than any one result by itself. But no general rule can be given as to how far it is advisable to continue repeating observations. There are certain errors called "*systematic errors*," which are not got rid of by any number of repetitions; thus if a centimetre rule is not correct, a length measured by means of it will be wrong by a certain amount, however often the measurement is repeated. It will clearly be useless to diminish "accidental errors" beyond the limit at which we have reason to think "systematic errors" begin.

Repetition of observations.—As it is often one of the most difficult problems of experimental work to determine the relative importance of systematic and accidental errors, each exercise will contain instructions when a repetition of observations seems necessary; but students should understand that, according to the accuracy of the result which is required, it may sometimes be necessary to repeat the observations more often than they are asked to do, and in other cases, where instructions are given to repeat an experiment, a single one may be sufficient if only a rough result is required. What is aimed at in this book is a fair result, so far as the apparatus used allows it to be obtained without exceptional skill or trouble.

By trying in the first instance to understand the reason for these instructions, and by following in other cases the advice given them by their teachers, students will gradually gain sufficient experience to be able to exercise their own judgment in the disposition and conduct of an experiment. The training of that judgment is one of the principal objects of a course of Practical Physics.

SECTION II

Arithmetical Calculations

THE arithmetical calculations which occur in physical work can often be shortened by methods which students will find it advisable to practise. The most useful of these methods is the process called shortened multiplication, which may be applied when the product of two numbers is required within certain limits of accuracy only. Thus let a diameter of a circle be measured with a centimetre scale, and its length found to be 2·14 cms. We shall assume that the result is correct to the nearest millimetre, but that different measurements result in slightly different values, showing that there may be an error in the measurement amounting to ·01 cm., or to one half per cent. of the length measured. If the student is required to calculate the circumference of the circle from the measured diameter, he will have to multiply 2·14 by $\pi = 3·14159$, but however many figures in π he takes into account, nothing can alter the fact that his result will be wrong by one half per cent., if the diameter is measured wrongly to that extent. It follows that it will be justifiable to neglect all figures of π after the third, and to multiply therefore 2·14 by 3·14. These two numbers multiplied together give 6·7196, and the possible error which it is known may affect the result, renders the second decimal place doubtful

The third and fourth decimal places are not only doubtful, but they possess no value at all, the chances against their being correct being exceedingly great. Hence to include them would not only give no additional accuracy, but would be absolutely misleading. It would save time therefore, without any diminution of accuracy, if we could obtain the first three figures of the above product without the last two, and this we are able to do by means of shortened multiplication, which we now proceed to explain.

Suppose that it is required to multiply 7239 by 5826, and that it is known beforehand, that owing to the possible errors in these numbers, it is useless to find the result to more than four figures. The customary form of multiplication would be :—

```
    7239
    5826

   43434
  14478
 57912
36195
--------
42174414
```

It would be just as simple, however, to begin by multiplying by the first figure on the left-hand side of the multiplier—in this case 5. This would be better in all cases, because the most important part of the result would be obtained first. The work would now run as follows :

```
 7239
 5826

36195|
 5791|2
  144|78
   43|434
-----|---
42174|414
```

If the result is wanted to five figures only, everything that stands on the right-hand side of the vertical line drawn above, is useless and need not be written down. Thus the rule for shortened multiplication will be:—Begin multiplying by the left-hand figure of the multiplier, writing out the product completely. Next take the second figure of the multiplier, but in the calculation omit to take any notice of the right-hand figure of the multiplicand, and place the first figure of the product directly underneath the right-hand figure of the first line. Next take the third figure from the left of the multiplier, and commence with the third figure from the right of the multiplicand, and so on. It will help the beginner, in practising this form of multiplication, to make a stroke through the figures of the multiplicand as they are no longer required The successive steps of the calculation will be as follows:

7239	723~~9~~	72~~39~~	7~~239~~
5826	5826	5826	5826
36195	36195	36195	36195
	5784	5784	5784
		144	144
			42
			42165

The line through the 9 of the multiplicand is drawn when the multiplication by the 5 is performed, in order to indicate that in multiplying by 8, it should no longer be taken into account, and as successive products are formed the successive figures of the multiplicand are struck out. In the final result the last figure may be inaccurate by a few units, as will be seen by comparing the above example with the complete calculation. The last figure is therefore omitted in writing down the result, but if it is 5 or greater than 5, the next figure on the left is increased by 1.

Multiply by shortened multiplication:

2587	×	6235
4921	×	3857
8467	×	1304
85928	×	6005

The accuracy of the last figure obtained in shortened multiplication, may be improved by performing mentally the multiplications to one more figure than is required; that is beginning in each case with the figure last struck out, and including in the first figure written down, the number which is "carried."

In the following examples multiplication is performed on the left hand side, by the process previously described, while on the right, the numbers carried over are taken into consideration.

```
 568        568
 464        464
----       ----
2272       2272
 336        341
  20         22
----       ----
2628       2635
```

The complete result is 263552, and it is seen that a material advantage has been gained by the improved process. But students should not try to apply this method of obtaining increased accuracy before they have become thoroughly familiar with the simpler process.

Division may be shortened in a similar manner.

The following example will explain the method. Students should write out the division in the ordinary way, and compare the complete and shortened forms.

Divide 7925 by 2693, the result to be correct to four figures.

Shortened form:

```
2693 ) 7925 ( 29429
       5386

       25390
       24237
       -----
        1153
        1076

          77
          52
       -----
          25
```

The result to four figures would be 2·943, the position of the decimal point being obvious. It will be seen, that for the first two figures of the quotient, the division is carried on as usual, but after that, instead of adding a 0 to the remainder, and multiplying the complete divisor by the next figure of the quotient, the last figure of the divisor is omitted, in the next step the last two figures are omitted, and so on. As in multiplication, we may obtain more accurate results by taking account of the number carried from the first of the omitted figures in the divisor.

The example worked out in this way would be:

```
2693 ) 7925 ( 29428
       5386
       -----
       25390
       24237

        1153
        1077
       -----
          76
          54

          22
```

which is correct to five figures.

The position of the decimal point is always best ascertained by carrying out an independent calculation to one or two figures only.

Thus if the value of $\frac{42\cdot7 \times \cdot0296}{\cdot000528 \times 6370}$ in the form of a decimal were required, a rough approximation to the result would first be found by calculating out

$$\frac{40 \times \cdot03}{\cdot0005 \times 6000} = \frac{1\cdot2}{3} = \cdot4$$

Next the value of $\frac{427 \times 296}{528 \times 637}$ irrespective of the decimal place would be calculated. The number obtained after multiplication and division to three figures is 376, and the approximate value previously obtained fixes the decimal point, so that the final result may be put down as ·376.

A simplification of the arithmetical work is often possible when large and small quantities enter together into the calculation. Let, for instance, the product of $1 + \delta$ and $1 + \epsilon$ be required, δ and ϵ being so small compared to unity, that their product may be neglected. The complete product would be $1 + \delta + \epsilon + \delta\epsilon$, and neglecting the last term we have :—

$$(1 + \delta)(1 + \epsilon) = 1 + \delta + \epsilon \text{ (approximately).}$$

More generally, if δ and ϵ are both small, so that $\delta\epsilon$ is negligible compared to ab,

$$(a + \delta)(b + \epsilon) = a\left(1 + \frac{\delta}{a}\right)b\left(1 + \frac{\epsilon}{b}\right) = ab\left(1 + \frac{\delta}{a} + \frac{\epsilon}{b}\right)$$
(approximately).

The following equations, which are often useful, will hold whenever δ^2/a^2 is a negligible quantity:

$$(a + \delta)^2 = a^2 + 2a\delta = a^2\left(1 + \frac{2\delta}{a}\right)$$

$$(a - \delta)^2 = a^2 - 2a\delta = a^2\left(1 - \frac{2\delta}{a}\right)$$

$$(a + \delta)^3 = a^3 + 3a^2\delta = a^3\left(1 + \frac{3\delta}{a}\right)$$

$$(a - \delta)^3 = a^3 - 3a^2\delta = a^3\left(1 - \frac{3\delta}{a}\right)$$

$$\frac{1}{a + \delta} = \frac{a - \delta}{a^2} = \frac{1}{a}\left(1 - \frac{\delta}{a}\right)$$

$$\frac{1}{a - \delta} = \frac{a + \delta}{a^2} = \frac{1}{a}\left(1 + \frac{\delta}{a}\right)$$

$$\sqrt{a \pm \delta} = \sqrt{a} \pm \frac{\delta}{2\sqrt{a}} = \sqrt{a}\left(1 \pm \frac{\delta}{2a}\right)$$

All these are included in

$$(a \pm \delta)^n = a^n\left(1 \pm n\frac{\delta}{a}\right)$$

As an example we may take a case which will be useful when barometric pressure has to be accurately determined. It will be shown (Section XII.) that a certain correction has to be applied to the barometer reading, depending on the temperature of the mercury column of the barometer. This correction is $-aht$, where a is a known number, h the observed height of the barometer, and t the temperature. Under ordinary circumstances the height of the barometer will be approximately 76 centimetres, and the temperature not very far from 15°C., we may therefore write:—

$$h = 76 + \kappa$$
$$t = 15 + \delta$$

where κ and δ are small numbers and their product $\kappa\delta$ therefore small compared to ht, and as the total correction is small, it is generally sufficient to neglect $\kappa\delta$.

Applying the previous equations we find

$$ht = (76 + \kappa)(15 + \delta) = (76 \times 15) + 15\kappa + 76\,\delta \text{ (approximately).}$$

If for κ and δ we write again $h-76$ and $t-15$ respectively, we find

$$ht = 1140 + 15\,(h - 76) + 76\,(t - 15)$$

and if the equation is multiplied by a, the numerical value of which is $\cdot000163$, we obtain

$$aht = \cdot186 + \cdot0024\,(h - 76) + \cdot0124\,(t - 15^\circ)$$

Although the right-hand side of this equation looks more complicated than the left-hand side, $h - 76$ and $t - 15$ will be small numbers, and the products to be calculated will be obtained without trouble, especially if shortened multiplication is used.

The arithmetical mean of two numbers a and b is defined as $\frac{a+b}{2}$; the geometrical mean is defined as $\sqrt{ab}$. The arithmetical mean is always greater than the geometrical mean, for if twice the geometrical mean is taken from twice the arithmetical mean, the remainder is equal to $(\sqrt{a} - \sqrt{b})^2$ which is always a positive quantity. If a and b differ only by a small quantity, we may put $b = a + \delta$, and by applying the equation given on p. 15 we find

$$\begin{aligned}\sqrt{ab} = \sqrt{a^2 + a\delta} &= a\sqrt{1 + \frac{\delta}{a}} \\ &= a\left(1 + \tfrac{1}{2}\frac{\delta}{a}\right) \\ &= \tfrac{1}{2}(a + b)\end{aligned}$$

The last equation shows that if two quantities a and b are so nearly equal, that the square of $(a - b)$ can be neglected in comparison with a^2, the geometrical mean may be taken to be equal to the arithmetical mean.

Example I.—If $a = 2$, $b = 3$ $\delta = \cdot 1$ $\epsilon = \cdot 1$ calculate approximately the value of $(a + \delta)(b + \epsilon)$, and find by how much the result differs from the one which is strictly accurate. If the product had been required to one per cent., would it have been sufficient to use the approximate method?

Example II.—Calculate to two figures the value of

$$\frac{(113{\cdot}78)^3 - (113{\cdot}63)^3}{(113{\cdot}63)^3}$$

Example III.—Calculate $\frac{1}{\sqrt{\cdot 998}}$ by the approximate method, and find how nearly the result is correct.

The use of logarithms is a great help in many arithmetical calculations, and it will be an advantage to the students, if simultaneously with the present course in practical work, they obtain instruction in multiplication and division by logarithms. Tables giving logarithms to four decimal places will be quite sufficient for the purpose.

SECTION III

Graphical Constructions

In physical problems mathematical work is often replaced with advantage by graphical constructions. We shall illustrate the principle and use of such a graphical representation by an example. Let the connection between the scales of two thermometers, one (A) graduated on the Centigrade scale and reading correctly, the other (B) graduated according to some unknown scale, be determined experimentally and exhibited in a graphical form. In the first place, a number of observations are taken as described in Section XVI., by plunging the two thermometers side by side into water at different temperatures; a number of readings of A, corresponding to an equal number of readings on B, are thus obtained. These observations are to be represented by a curve. Draw two lines (Fig. 4) at right angles to each other, OX horizontal and OY vertical, each of these two lines being called an axis. Imagine OX to be graduated in Centigrade degrees, and OY in degrees of the other scale. If now for instance, A reads 30° when B reads 24°, take a point marked 30 on the scale along OX and through it draw a line parallel to OY. Also take the point 24 on the scale of OY, and through it draw a line parallel to OX. These two lines

will intersect at a point P. A similar comparison might shew that 50° on A corresponded to 36° on B, and a point Q could thus be found exactly in the same way as P. For each observation one point is obtained, and when a sufficient number of observations have been taken, all the points may be united by a curve. In this case if both thermometers have been correctly graduated the curve will be a straight line MN. If MN cuts OY

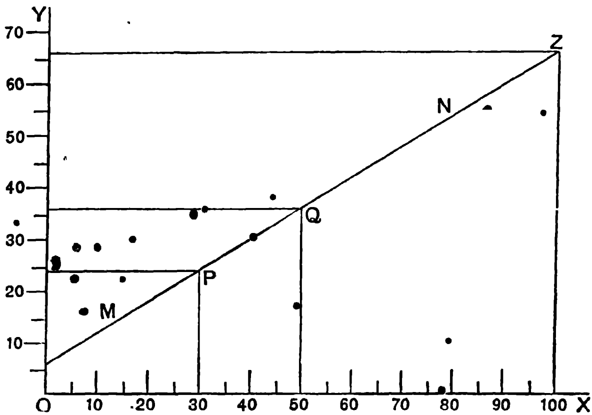


FIG. 4.

at the point 6 of the scale, it will show that the freezing point on the thermometer B is at division 6 of its scale. To find by graphical construction the position of the boiling point on the thermometer B, in other words the point corresponding to 100° on the Centigrade scale, we must draw a vertical line through that point on OX which reads 100; if this line intersect MN in Z, and if through Z a horizontal line be drawn, this horizontal line will be found to intersect OY at a point

corresponding to the reading 66. Hence the boiling point on B lies at 66. The thermometer B has therefore been graduated so that its freezing point is at the division 6, and that 60 degrees of its scale correspond to 100 degrees on the Centigrade scale.

An important point to be attended to in graphical constructions, is the fixing of the scales according to which the axes are to be divided, as the best scale to be used can only be determined in each particular case. It is often necessary to take very different scales along the two axes. The quantity to be measured along OY may, for instance, be the error of a thermometer, and never exceed 0·1 degree, while the axis of OX may have to include all the points of the thermometric scale between the freezing and the boiling points. In that case, one centimetre along OX may be taken to represent 10°, while one centimetre along OY may represent only 0°·01.

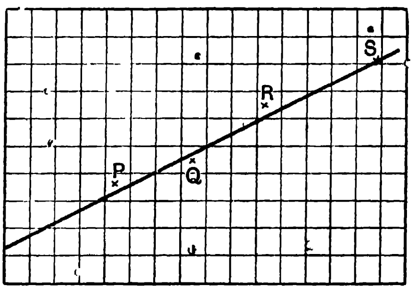


FIG. 5.

The graphical method may also be used occasionally to correct errors of observations. Thus suppose that in a comparison of two thermometers, the points P, Q, R, S (Fig. 5) have been obtained, which, it is seen, do not all

lie exactly in a straight line. A straight line may be drawn however in such a way that the points lie as near to it as possible, and that straight line will probably more nearly represent the relation between the thermometric scales, than a curve which actually passes through the points.

Considerable labour is saved if the paper on which curves are plotted is "squared paper," *i.e.* is divided into squares as shown in Fig. 5.

SECTION IV

Units

ALL measurements in this volume are referred to the so-called metrical system. The standard of length in this system is the metre,[1] which was originally chosen because it was supposed to be one ten-millionth of the distance between the earth's equator and the pole. Practically however the metre is the distance between two marks on a certain platinum rod kept at Paris, when the temperature of the rod is 0°C. The metre is subdivided as follows:

$$\begin{aligned} 1 \text{ metre} &= 10 \text{ decimetres} \\ &= 100 \text{ centimetres} \\ &= 1000 \text{ millimetres.} \end{aligned}$$

Also

10	metres	are	called	1 decametre
100	,,	,,	,,	1 hectometre
1000	,,	,,	,,	1 kilometre.

[1] There is at present no uniformity in the English spelling of the metric units, mètre, metre, and meter being adopted by different writers. There is a certain convenience in distinguishing the *metre* as a measure of length, from the word *meter* which is in use to indicate certain instruments as the gas *meter*, water *meter*, or *micrometer*. Hence we have adopted the above spelling, which is also the most usual one. But there is no similar reason for keeping to the French gramme, and we shall use the spelling gram instead.

The relation between the English and metric units of length, to one part in a million is :—

$$1 \text{ metre} = 3\cdot28090 \text{ feet.}$$

From this relation any measurement of length taken in one system of units may be reduced to the other, but the following additional relations, which may be deduced from the one given, are often useful. They are of course correct only to the last figure given.

1 metre	= 39·37	inches
1 centimetre	= 0·3937	,,
1 inch	= 2·5400	centimetres[1]
1 foot	= 30·48	,,
1 yard	= 91·44	,,
1 mile	= 1·609	kilometres
1 kilometre	= 0·621	miles
8 kilometres	= 5	miles nearly.

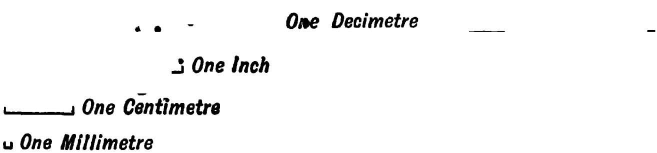


FIG. 6.

Students should make themselves familiar with the lengths of a metre, a centimetre, and a millimetre. It is a very useful exercise to estimate by eye a certain length in centimetres or millimetres, and then test the estimate by actual measurement. For this purpose a number of lines should be drawn at random on a sheet of paper, their length varying from a few millimetres to ten centimetres, and an eye estimate made of each length, the

[1] Note the zeros at the end (*see* page 4).

estimate put down by the side of each line, and each line then be measured with a centimetre rule, and the error made in each case determined. Fig. 6 shows the lengths of a decimetre, an inch, a centimetre, and a millimetre.

The relations of the units of area and of volume may be obtained from those of length.

1 square centimetre	= 100	square	millimetres
1 ,, decimetre	= 100	,,	centimetres
1 ,, metre	= 100		decimetres
	= 10000		centimetres
	= 1000000		millimetres
1 cubic centimetre	= 1000	cubic	millimetres
1 ,, decimetre	= 1000	,,	centimetres
1 ,, metre	= 1000	,,	decimetres
	= 1000000	,,	centimetres.

1 cubic decimetre is called a litre.

Sufficient data have already been given to deduce the relations between the English and metric systems, but the following are put down for the sake of reference.

1 square centimetre	= 0·1550 square inches
1 ,, inch	= 6·451 ,, centimetres
1 ,, yard	= ·8361 ,, metres
1 acre	= 4840 ,, yards
	= 4047 ,, metres
1 cubic centimetre	= ·0610 cubic inches
1 litre	= 61·03 ,, ,,
1 cubic inch	= 16·39 ,, centimetres
1 ,, foot	= 28·315 litres
1 litre	= 1·76 pints
1 pint	= 567·9 cubic centimetres
1 quart	= 1·136 litres
1 gallon	= 4·543 litres.

Note and *explain* in your note-book why the number of litres given as being equal to one gallon is not, as far as the last figure is concerned, accurately equal to four times the number of litres stated to be contained in a quart.

The metric standard of mass, the *kilogram*, is the mass of a certain piece of platinum kept at Paris and is intended to be the mass of 1 cubic decimetre of water at 4° C.

1 kilogram	=	1000 grams
1 gram	=	10 decigrams
1 decigram	=	10 centigrams
1 centigram	=	10 milligrams.

The expressions decagram, and hectogram for 10 and 100 grams respectively are not much used.

1 kilogram	=	2·2046 lbs. avoirdupois
1 gram	=	15·4 grains
1 lb. avoirdupois	=	453·6 grams
1 ounce ,,	=	28·35 grams
1 grain	=	64·8 milligrams.

The following abbreviations are in common use:

cm	stands	for	centimetre
mm	,,	,,	millimetre
cc or cub. cm.	,,	,,	cubic centimetre
gm or grm	,,	,,	gram
mgm	,,	,,	milligram.

When great lengths are measured, it is usual to express them in kilometres, while small lengths are expressed in centimetres and millimetres, since it would obviously be inconvenient to have only one unit to express, for instance, both the distance of two towns apart, and the size of a small microscopic object. In every case therefore in which a measurement is given, the unit in which it is expressed should be clearly stated. But the unit of length enters also into the measurement of other quantities; thus the number expressing a certain velocity, or a pressure, or an energy, would be different according to whether the inch, the centimetre, or the metre were taken as the unit of length. To avoid confusion in these cases, the *centimetre* is always taken as the unit. Similarly the gram is taken as the unit of mass, and the second as the unit of time. Numbers which are referred to these units are said to be expressed in the C.G.S. (centimetre-gram-second) system.

PART II

MECHANICS

SECTION V

The Vernier

Apparatus required.—Two wooden models of verniers, sliding calipers, block of wood, metal cylinder, barometer vernier, angular vernier.

If the length of a body AB (Fig. 7) is to be measured by means of a graduated scale, it will generally happen that, when the end A is placed opposite the zero of the scale, the end B falls somewhere between two divisions. Thus in fig. 7 we should say that the length AB was between four and five units of the scale, and we might even judge by estimation that it was about 4·3 units. If the mere judgment of the eye is not to be relied on to subdivide each scale division but a more accurate method is required, a contrivance called the Vernier may be used.

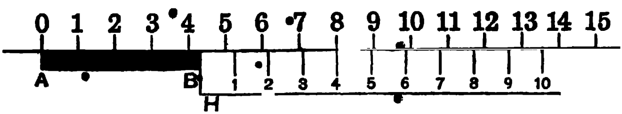


FIG. 7.

Let a small scale called the Vernier Scale HK, which is divided so that ten of its divisions equal in length nine divisions of the principal scale, be placed against the

end B of the body to be measured. Looking along the two scales we see that generally their divisions do not coincide, but that the division marked 4 on the Vernier scale is in the same line with a division marked **8** on the principal scale. This fact will at once enable us to say that the length AB is 4·4 units of the principal scale, for since 10 divisions of the Vernier scale = 9 divisions of the principal scale, 1 division of the Vernier scale = ·9 divisions of the principal scale, or a Vernier division is less than a scale division by ·1 of a scale division. Hence the part of AB which extends over division **4**, *i.e.* the distance between **4** of the principal scale and 0 of the Vernier, is greater than the distance between **5** of the scale and 1 of the Vernier by ·1 of a scale division. Thus :—

Distance between **4** and 0	= distance between		**5** and 1 + ·1	scale div.
	=	,,	,,	**6** and 2 + ·2	,,
	=	,,	,,	**7** and 3 + ·3	,,
	=	,,	,,	**8** and 4 + ·4	,,

Or the total length of AB = 4·4 scale divisions.

The student will easily see in the same way, that if the eighth division of the Vernier scale instead of the fourth had been coincident with one of the divisions of the principal scale, the length would have been 4·8; and generally after the whole unit has been read off on the principal scale, the decimal place, or even sometimes more than one decimal place, may be found by *reading that division on the Vernier scale, which coincides with one of the divisions of the principal scale.*

Exercise I

The Vernier "A" provided has its divisions of such a length, that ten of them are equal to nine of the principal scale. The student should explain in his notebook, how this Vernier may be used for measurements of length, and measure by it the length, breadth, and thickness of the small block of wood provided.

Before making each measurement, it is advisable to estimate the length to be measured as accurately as possible by eye, the estimation being made by supposing a division on the scale to be subdivided into ten equal parts.

Record as follows :—

10 Vernier divs. = 9 scale divs.
1 " " = ·9 " "
∴ 1 scale div. exceeds 1 Vernier div. by ·1 scale div.

Measurement of length of block No. 5 :

Eye estimate of length .	4·3
Reading on principal scale .	= 4·0 scale div.
Coincidence at 4th div. of Vernier =	·4 " "
Total .	4·4 " "

Similarly for breadth and thickness.

Exercise II

The Sliding Calipers

Examine the Vernier on the centimetre scale of the sliding calipers provided, and by means of them measure the length of a brass cylinder (Fig. 8). The reading on

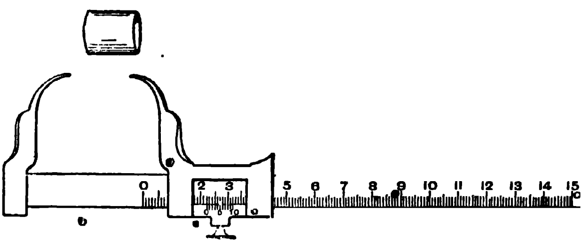


FIG. 8.

the scale when the jaws just touch each other, should first be taken, then the reading when the jaws just touch the two ends of the cylinder. The difference is the length required. Repeat the two measurements.

Record as follows :—

Reading with cylinder between jaws	2·21 - 2·22	mean	=	2·215 cm.
,, ,, jaws together . .	·02 - ·03	,,	=	·025 ,,
Length of cylinder No. 8				2·19 ,,

All Verniers are the same in principle, but in some cases the value of one Vernier division may not be so evident as it is in the simple case of which Fig. 7 is an example. Take, for instance, that illustrated in Fig. 9. There the principal scale is subdivided into half units, each fifth unit being marked. The length AB is evidently greater than 2·5, and less than 3 scale divisions, and the Vernier enables the excess over 2·5 to be determined. The Vernier scale contains 25 divisions, which are equal to 24 small divisions of the principal scale, so that the length of each Vernier

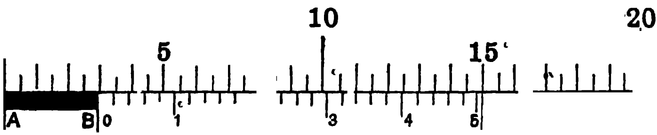


FIG. 9.

division is $\frac{24}{25} \times \frac{1}{2} = \cdot 48$ of the scale unit, or a Vernier division is ·02 less than a scale division. The coincidence is seen to take place at the seventeenth Vernier division, hence :—

Distance between 2·5 and 0 = distance between 3·0 and ·2 + ·02 scale units
= ,, ,, 3·5 and ·4 + ·04 ,,
= ,, ,, 4·0 and ·6 + ·06 ,,
= ,, ,, 4·5 and ·8 + ·08 ,,
&c.
= ,, ,, 11·0 and 8·4 + ·84 ,,
= ·84
Or the total length AB = 2·5 + ·84 scale units
= 2·84 ,, ,,

The meaning of the numbers opposite each fifth division on the Vernier scale is now clear If the coincidence

had taken place accurately at the point marked 3 on the Vernier, we should have had to add ·3 to the reading on the principal scale next below B, which is 2·5. The unit on the Vernier is divided into five parts, hence the value of each subdivision is ·02, and as the coincidence takes place at the second division from the point marked 3, we have to add ·34 to the reading of the principal scale. Reading a Vernier scale of this nature we should put down—

Eye estimate of length	2·9	scale units.
Reading on principal scale	2·5	,,
Reading of point of coincidence on Vernier.	·34	,,
Length AB	2·84	,,

Whenever a Vernier has to be read, the first step should be to make sure of the value of one division on each of the two scales.

Exercise III

The barometer which is suspended in the laboratory, has a scale on one side divided into inches, and on the other into millimetres. The Vernier on the inch scale is divided as in Fig. 9, the unit only being different.

The Vernier "B" provided, is a model of the barometer Vernier. Determine by means of it the dimensions of the block of wood previously measured, putting down the reasoning and results as in the case of the "A" Vernier above.

Examine the Verniers on both the metre and inch scales of the barometer, and explain how the former should be read.

The circular or angular Vernier is used in the same way to measure angles, *e.g.*, to minutes of arc, on a circular scale graduated in degrees only. Examine the model supplied and explain how it is read.

SECTION VI

The Spherometer and Screw Gauge

Apparatus required.—A spherometer standing on a glass plate, a brass cylinder, a brass plate, sliding calipers, a scale divided into centimetres and millimetres, a large lens, and a screw gauge.

When objects of small size have to be measured accurately, instruments depending on the motion of a screw, *e.g.*, the screw gauge or the spherometer, may be used.

Study carefully the construction of the spherometer provided (Fig. 10). Note that while the screw is turned

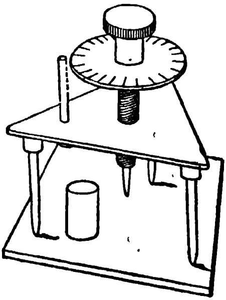


FIG. 10.

through a whole revolution, it is raised through a distance equal to the pitch of the screw, *i.e.* to the distance from thread to thread in a direction parallel to the axis of the screw. Note also that the circumference of the head is divided into 100 equal parts, so that decimal parts of a revolution can be read off.

EXERCISE I

To find the Pitch of the Screw of the Spherometer

A brass cylinder is provided, the length of which must be measured in the first instance with a millimetre scale, *and estimated to a tenth of a millimetre.* The length found thus by estimation should be verified by measurement with the sliding calipers.

In order to determine the relation between the length of the cylinder and the pitch of the screw of the spherometer, place the instrument on a glass plate. Turn the screw until its point just touches the glass plate. This is readily done, for if the screw is turned a little too far, the instrument may be made to rock by slightly tapping it above one of the legs. If therefore the point of the screw is too low to begin with, it can be gradually raised until the rocking ceases. When the point has been adjusted in this way, read off the division of the disc which is opposite the vertical bar. Several readings should be taken, the screw being re-adjusted each time. Next turn the screw so that its point is raised, counting the number of times the mark at the zero of the graduations on the head passes the vertical rod, until the brass cylinder can be introduced beneath the point. Adjust the screw again until the point is in contact with the top of the cylinder, repeating the operation and reading off each time the division on the disc opposite the rod.

Note.—In order to be able to count the number of complete revolutions readily, it is convenient to provide a white mark at the zero of the circular disc; this is easily seen while the screw is turned round, and each time it passes the vertical bar, a whole revolution is completed. Great care should be taken to obtain the number of revolutions without error.

(1) Take the mean of the readings when the point of the screw was in contact with the glass plate, (2) the number of complete revolutions through which the screw was turned, (3) the mean of the readings when the point was in contact with the top of the cylinder. Add (3) with a decimal point prefixed to (2), and subtract from the sum (1) with a decimal point prefixed. The difference is the number of turns and decimal parts of a turn equivalent to the length of the cylinder. Divide that length by the number of turns, the quotient is the pitch of the screw. The observations and results should be recorded as follows:—

Spherometer No. 12.

Reading for top of cylinder	37 turns 46 divs	=	37·46 turns
,, ,, glass plate	0 ,, 58 ,,	=	·58 ,,
Difference			36·88 ,,

Length of cylinder No. 10 = 18·5 mms.

$\therefore$ Pitch $= \dfrac{18{\cdot}5}{36{\cdot}88} = {\cdot}498$ mms.

EXERCISE II

To Measure the Thickness of a Brass Plate by means of a Spherometer

Proceed as in the previous exercise, substituting the plate for the cylinder, and finding the number of turns of the screw equivalent to the thickness of the plate. This number multiplied by the pitch gives the thickness of the brass plate. Take the mean of two sets of measurements.

Record as follows :—

Plate No. 11.

Reading for top of plate	2 turns 81 divs.	=	2·81 turns
,, ,, glass plate	0 ,, 58 ,,	=	·58 ,,
Difference			2·23 ,,

Thickness $= 2{\cdot}23 \times {\cdot}498 = 1{\cdot}11$ mms.

EXERCISE II*a*

To Measure the Radius of Curvature of a Surface by the Spherometer

Place the spherometer on the glass plate and determine the zero reading, then place it on the surface and determine the reading in the same way. If h is the difference of the readings reduced to cms., r the distance from the point of the screw to the ends of the legs of the spherometer, and R the radius of curvature of the surface, then

$$R = r^2/2h.$$

r is most conveniently determined by placing the spherometer on a sheet of paper with the point of the screw in contact with the paper and pressing it so that impressions of the ends of legs and screws are produced. The measurement can then be made on the paper. Record as in the previous Exercise.

EXERCISE III

Use of the Screw Gauge

The screw gauge (Fig. 11), is the same in principle as the spherometer, and the pitch of the screw may be determined as in Exercise I. If it is only required approximately, unscrew the head, noting that at each revolution it passes one division of the scale on the stem. When the head has

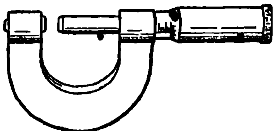


FIG. 11.

been screwed off sufficiently to leave about 2 cms. of the scale on the stem exposed, compare the scale with a millimetre scale, and thus find in millimetres the value of a scale division, *i.e.* of the pitch of the screw. Having found the pitch, screw the gauge up, using the finger and thumb to gently rotate the screw till the jaws are just in contact, which can be detected by the slight increase of resistance to the rotation. Read the last division visible on the stem, and the division on the head opposite the mark on the stem from which the stem divisions are drawn. This is the zero reading. Now insert the body to be measured, screw up gently till the jaws just touch it, and take again the readings on stem and head. From the pitch and these measurements, determine the thickness of the body measured.

Find by the gauge the thickness of the brass plate previously measured, and record as follows:—

Screw Gauge No. 2.

Pitch found = 1 mm.

Reading when Plate 11 between jaws	1·12 mms.
,, ,, jaws together . .	·01 ,,
Thickness of Plate No. 11 .	**1·11 mms.**

SECTION VII

The Law of Moments

Apparatus required.—Moments apparatus, and weights.

Definition.—If P (Fig. 12) be any force and O a point, then if from O a perpendicular to the direction of P be drawn, and p be the length of this perpendicular, Pp is the moment of the force P with respect to the point O.

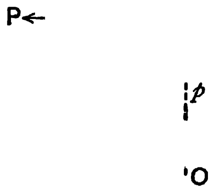


FIG. 12.

It has been found convenient to call the moment *positive*, if P tends to produce rotation about O in a direction *contrary to that of the hands of a watch*, and negative if with the hands of a watch. Thus if, in Fig. 13, P′ be the magnitude of a second force irrespective of sign, the moment of P′ with respect to O will be $-P'p$.

Proposition.—It is proved in books on Mechanics that if a body acted on by forces in one plane only is in equilibrium, the algebraic sum of the moments of the

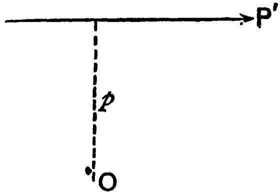


FIG. 13.

forces with respect to any point in their own plane vanishes; or in other words, the sum of the moments having positive signs, must be equal to the sum of the moments having negative signs.

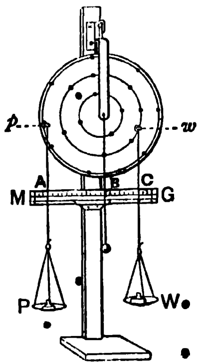


FIG. 14.

EXERCISE

To verify the law of moments in the special case of two parallel forces, a balanced disc of wood capable of rotating

about its centre is provided (Fig. 14). Pans in which weights can be placed are suspended from holes bored in the disc.

See that the disc is vertical, moves freely about its axis without touching its supports, and will come to rest in any position, *i.e.*, is in neutral equilibrium.

Proceed as follows :

(1) Weigh together the scale pan P and the string and peg *p*, also the pan W and the string and peg *w*.

(2) Place the pegs in two holes that lie on the same diameter of the disc and at equal distances on opposite sides of the centre, and suspend the pans P and W from the pegs. Notice that if the weights of the pans and contents are equal, the disc is in *indifferent* equilibrium, and will remain at rest in any position.

(3) Remove one of the pegs with the attached pan, and place it in a hole such that the line joining the two pegs does not pass through the centre of the disc. Place weights in P and W, and notice that the disc will come to a position of *stable* equilibrium, if the line joining the pegs passes *under* the centre, but that the equilibrium will be *unstable*, if the line passes *over* the centre of the disc.

(4) In the case of stable equilibrium, read in the glass scale MG the positions of the strings at A and C, and of the plumb line in the middle at B. In reading the position of each string, place the eye in such a position that the string covers its own image in the mirror (*see* p. 6, "*Parallax*"). Read both edges of the strings and take the mean.

(5) Alter the positions of the pegs, and the weights in the pans, and take fresh readings.

(6) Reduce the observations as follows: If P_1 is the weight placed in the pan the weight of which and accessories is P, and if p is the perpendicular between the string from which P is suspended and the string at B; the

moment of $P+P_1$ about the axis of rotation is $(P+P_1)p$. This must be equal to the corresponding quantity on the other side. Hence :—

$$(P + P_1)p = (W + W_1)w$$

$$\text{or :—} \quad \frac{P + P_1}{W + W_1} = \frac{w}{p}$$

The ratio of the weights $P+P_1$ and $W+W_1$ can be found with great accuracy, but the difficulty of the experiment lies in the accurate determination of the perpendiculars. It was found in one experiment, for instance, that when the ratio of the weights was 1·187, the inverse ratio of the corresponding perpendiculars was only 1·165. The unavoidable errors of observation combined in this case therefore, to produce a difference of two per cent. between two numbers which should be equal. The difference is not more than was to be expected from the necessarily rough method of measuring the perpendiculars. If the measurement of one of these was one per cent. too large, and that of the other one per cent. too small, the error would be accounted for. Owing to the friction of the axle of the disc in its bearings, the position of equilibrium is a little uncertain, which will also tend to produce an error.

Enter observations and results as follows :—

Moments Apparatus No. 6.

Weights in grams.					Distances in cms.				
Left pan=30·0		Right pan=30·5		Ratio left/right	Plumb line at 0		Ratio right/left	Differ.	Error %
In left	Total	In right	Total		To left	To right			
50	80	30	60·5	1·32	9·70	12·9	1·33	– ·01	– ·7
80	100	50	80·5	1·24	10·2	12·7	1·25	– ·01	– ·7
		&c.				&c.			

SECTION VIII

The Pendulum

Apparatus required.—A simple pendulum with a wooden rod graduated in centimetres behind it, on which two millimetre scales engraved on mirrors slide; also a watch with a seconds hand.

The relation between the length l of a simple pendulum, its time of oscillation t, and the value of the gravitational acceleration g, is proved in books on Mechanics to be

$$t = 2\pi\sqrt{\frac{l}{g}}.$$

and is independent of the material of the bob.

If, therefore, the time of oscillation of a simple pendulum of known length is observed, the value of g may be found. The above equation gives:—

$$g = \frac{4\pi^2 l}{t^2}$$

where $\pi^2 = (3{\cdot}1416)^2 = 9{\cdot}870$ and $4\pi^2 = 39{\cdot}48$ nearly.

Note.—By the "period" or "complete time of oscillation" is meant the time which a pendulum takes to go through the whole cycle of motion. If the time is reckoned, for instance, from the instant of passage of the pendulum through its position of equilibrium from left to right, the first oscillation is complete only when the pendulum again swings through its position of equilibrium from left to right.

Exercise

To determine the Value of the Gravitational Acceleration

In the apparatus provided, a leaden bullet is suspended by means of a string in front of a support graduated in centimetres, on which two glass millimetre scales with silvered backs slide (Fig. 15). The length of the pendulum can be ascertained as follows: The two glass scales are placed behind the point of suspension and the leaden bullet respectively, in such a way that when viewed normally (*see* "*Parallax,*" p. 6) the centimetre divisions on them coincide with the centimetre lines on the support. The eye should be placed so that the leaden bullet covers its own image in the glass scale behind, and the positions of its highest and lowest points read off, centimetres being read on the scale on the support seen through a strip of the mirror from which the silvering has been removed, and millimetres and tenths on the glass scale. The mean of the two readings gives the point on the scale which is at the same level as the centre of the bullet. The position of the upper end of the pendulum should be read in the same way. The length of the pendulum is the difference between the readings. Three experiments should be made, the lengths of the pendulum being approximately 80, 60, and 40 cms.

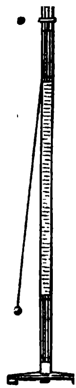


FIG. 15.

To find the time of oscillation: Place the clamp from which the string is suspended, so that the slit separating the jaws runs at right angles to the direction in which the pendulum is to swing. Draw the bullet aside (about 5 cms. for the greatest, and half that for the smallest length), holding it in the displaced position, and observing the

motion of the seconds hand of a watch held in the hand just below the bullet, so that both watch and bullet can be seen at the same time. As the seconds hand passes a given point (say 60) let the bullet go gently, without giving it an impulse in any direction. Count one each time the pendulum comes back to its original position, until it has performed 100 complete oscillations. Be careful to count 0 and not 1 as the bullet leaves the hand. When the proper number is nearly complete, watch again the seconds hand, and note down accurately the second at which the number of oscillations is complete. Having observed whether any entire minutes have elapsed since the beginning of the counting, and if so how many, write down in your note-book the number of seconds the pendulum took to perform 100 oscillations. If the pendulum, during its motion, changes its *plane* of oscillation considerably, so that there is danger of its striking against the support, it is a sign that the above directions for starting the pendulum have not been sufficiently attended to, and the observation must be repeated. The time observation should be taken twice for each length of the pendulum.

The observations and results are entered as follows :—

	I	II	III
Reading of point of support of string	·50	·50	·50
,, top of bob	82·92	55·51	39·50
,, bottom of bob	84·52	57·11	41·10
,, centre of bob b	83·72	56·31	40·30
Length of pendulum $l=(b-a)$	83·22	55·81	39·80
Time of 100 oscillations	184 183	148 150	126 127
Mean time of one oscillation t	1·835	1·490	1·265
t^2	3·367	2·220	1·600
l/t^2	24·81	25·14	24·88
$g = 4\pi^2 l/t^2 =$	979	992	983
Correct value	981		
Error	−2	+11	+2
Percentage error = Error /981	−·2	+1·1	+·2

Note.—Observe that any error in obtaining t will lead to double the percentage error in g, since in calculating g the value of t is squared. Hence, in order that the error should not exceed one per cent., the time of 100 vibrations should be correct to about half a second. This cannot be secured with a watch having an ordinary seconds hand unless several observations are made and the mean of the results taken.

SECTION IX.

The Hydrometer

Apparatus required.—A Nicholson hydrometer, a box of weights, a piece of glass, a piece of wax, and some salt solution.

EXERCISE I

To find the Specific Gravity of a Solid unacted on by a Liquid of known Specific Gravity.

The mean specific gravity of a substance is the ratio of its weight in vacuo (or in air so far as the present work is concerned) to the weight in vacuo of an equal volume of water.

(1) *To weigh the solid.*

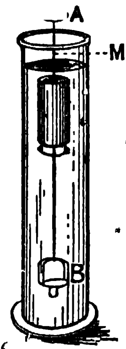


FIG. 16.

Fill the hydrometer jar with the liquid which we may take to be water, and place the hydrometer provided in the liquid. If air bubbles collect on the hydrometer remove them by touching them with the end of a wire, or the observations will be vitiated. Place weights w_1 in the pan A of the hydrometer (Fig. 16), until it sinks to the fixed mark M. Arrange the weight on the pan so that the hydrometer remains vertical. In using the weights avoid placing them on the table; a weight should always be either in the pan or in the box. Next remove the weights, place the solid in the pan A, and add weight w_2 till the hydrometer again sinks to M. The difference $w_1 - w_2$ between the weights required in the two cases, will be the weight of the solid.

(2) *To find the weight of an equal volume of water.*

Remove the weights from the pan A, and place the solid in the pan B if it is heavier, and in the cage under the float of the hydrometer if it is lighter than the liquid,

taking care that when the hydrometer is placed in the liquid there is no air bubble under the solid or attached to the hydrometer. More weight will now be required to sink the hydrometer to M, since the pressure of the fluid acting on the solid produces a resultant upwards, which, by the laws of hydrostatics, is equal to the weight of the fluid displaced by the solid, *i.e.* to the weight of a volume of fluid equal to that of the solid. If w_3 is the weight required, the weight of a volume of the fluid equal to that of the solid is $w_3 - w_2$, and if σ is the specific gravity of the fluid, the weight of an equal volume of water will be $(w_3 - w_2)/\sigma$.

(3) *To calculate the specific gravity.*

The specific gravity ρ of a body being the ratio of its weight to the weight of an equal volume of water, is calculated by the formula :—

$$\rho = \frac{\text{weight of body}}{\text{weight of equal volume of water}}$$

Hence $\rho = \dfrac{w_1 - w_2}{w_3 - w_2} . \sigma$. The body should be taken large enough to make it possible to determine both $w_1 - w_2$ and $w_3 - w_2$ to an accuracy of 1 per cent.

Record as follows :—

Hydrometer 3, glass block 4, fluid water $\sigma = 1$.

		Difference.
Weight to sink hydrometer to mark w_1	= 4·65 gr.	
		∴ glass = 3·44 gr.
,, with glass in upper pan w_2	= 1·21 gr.	
		∴ fluid = 1·46 ,,
,, ,, lower pan w_3	= 2·67 gr.	
∴ Weight of equal volume of water	= $1·46/\sigma$ = 1·46	
∴ Specific gravity of glass No. 4	$= \dfrac{3·44}{1·46} = 2·36$	

Repeat the observations in the reverse order. If the results agree closely, take the mean as the final result. If not, do a third experiment and take the mean of the three results.

Determine in the same way the specific gravity of the piece of wax, placing it when in the liquid in the cage under the float.

Exercise II

To find the Specific Gravity of a Liquid

Pour out the water from the hydrometer jar, and replace it by the liquid (a salt solution) from the cistern provided. Find the weight required to sink the hydrometer to the mark M in the solution. Then remove the hydrometer from the solution, dry and weigh it. If the weight of the hydrometer is W, and P is the weight to be added when it is placed in the salt solution, $W+P$ is the weight of the liquid equal in bulk to the immersed portion of the hydrometer. Similarly if P' has been added when the hydrometer is placed in water, $W+P'$ is the weight of water equal in volume to a weight $W+P$ of the salt solution. The specific gravity of the solution will therefore be

$$\frac{W+P}{W+P'}.$$

Record as follows :—

Hydrometer No. 3. Weight = 29·85 grs.

Weight to sink it in water =	2·65 gr.	total = 32·50 gr.
,, ,, ,, solution =	4·60 ,,	,, = 34·45 ,,
∴ Specific gravity of solution =	34·35/32·50	= 1·06

Repeat the observations in the reverse order. If the two experiments lead to results which agree closely, take the mean as the final value. If they differ much do a third experiment and take the mean of the three.

[After the observations have been taken, the salt solution should be poured back into the cistern.]

SECTION X

The Balance. I

Apparatus required.—Balance, box of weights and brass cylinder.

Students are supposed to be familiar with the principles on which balances are constructed.

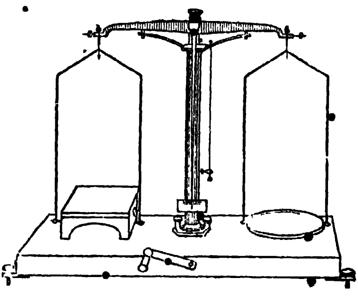


FIG. 17.

A balance (Fig. 17) is provided, by means of which the weight of a substance may be determined to the nearest centigram. The student should make a sketch of it in his note-book.

When the balance is not in use the beam is supported in a horizontal position by the two arms which project from the top of the pillar. and the under surfaces of the pans just touch the base board.

Through the centre of the beam a short triangular prism of steel or of agate passes, and one of the projecting edges of this prism constitutes the "knife edge" about which the beam turns when in use. Two shorter prisms of steel or of agate are placed at the ends of the beam, and the top edge of each prism furnishes the knife edge on which the stirrup which carries each scale pan is supported.

It is to preserve these knife edges that the beam of the balance is lowered when the instrument is not in use, so that as little weight as possible may rest on each knife edge.

The base screws allow the balance to be levelled. The beam and scale pans are brought into the proper position for use by gently moving the handle A at the base of the balance to the right. Notice as this is done that the two plane surfaces underneath the knife edges of the beam, are raised into contact with the knife edges, and the beam then raised from the supporting arms. As the beam rises the knife edges at its ends raise the pans stirrups, and finally the pans. When the beam is in its proper position the end of the long pointer attached to the centre of the beam is just in front of a horizontal graduated scale attached to the pillar of the balance. This scale should either be engraved on mirror glass, or have a small piece of mirror glass in contact with it behind.

Turn the handle A to the left so as to lower the beam. Make a sketch in your note-book of the way in which the knife edge at the end of the beam supports the pan stirrup.

When weighings are to be taken attend to the following instructions:

1. See that the balance swings freely when the beam is raised, and that when the pans are empty, the pointer attached to the beam oscillates equally or nearly so, on the two sides of the centre division of the scale. If not, screw out a little the small screw at the end of the beam on that side to which the pointer tends to move. This will slightly displace the centre of gravity of the beam, and the pointer may thus be made to occupy a position near the centre of the scale in front of which it moves. This position is called the "zero at no load."

2. As the scale is some distance behind the pointer, it is found that by moving the eye to right and left, the apparent position of the pointer on the scale alters owing to parallax (*see* page 6). To avoid errors due to this cause, always look at the pointer in the same direction, which can be secured if it is always made to cover its own image in the mirror behind the scale or on which the scale is engraved. It is not necessary that the pointer should stand exactly at the centre of the scale, so long as during a set of weighings it is always brought back to the same "zero."

3. Never place weights on the pans or remove them from the pans while the pans are raised above the base.

4. The contents of the boxes of weights are arranged so as to allow the correct weight to be quickly found by systematic trial. They consist of masses of 10, 5, 2, 2, 1 grams; and of 5, 2, 2, 1 decigrams and centigrams. If a box is incomplete the fact should be reported at once. In carrying out an observation, it will be found most convenient to begin by finding the *upper* limit to the weight required. Thus suppose the sub-

stance weighs 8·57 grams. On trial it will be found that 10 grams is too much. The next smaller weight 5 grams is not enough; the next weight 2 is added and is still not enough; the second 2 is added and is found too much; it is therefore replaced by the 1. Proceed in this way with the decigrams and centigrams, *trying each weight in descending order, and removing a weight from the pan only when it is found to be too much. When a weight is removed it should be replaced in its proper position, in the box, and not on the table.* Move the weights with the tweezers provided in the box, never with the fingers.

5. In order to see whether the body is properly balanced, it is not necessary that the beam of the balance should come to rest. It is only necessary to observe whether or not the oscillations are small and take place symmetrically about the "zero." The pointer should never be touched.

If a balance is always to weigh correctly, its arms, *i.e.*, the distance of the end knife edges from the central one—should be equal. As this is never absolutely the case, means must be taken to determine the inequality of the arms, and to find the true weight of the substance independently of the defects of the instrument.

Let a body of true weight P be placed in one pan of a balance, and let the arm on that side have a length a. Let the arm on the other side of the balance have a length b, and let W_1 be the weight on that side which will exactly balance P. Then by the law of moments

$$Pa = W_1 b.$$

If now P is placed in the other pan, it will require a different weight W_2 to produce a balance, and we have:—

$$W_2 a = Pb.$$

The first equation gives

$$\frac{a}{b}=\frac{W_1}{P} \quad . \quad . \quad . \quad (1)$$

and the second

$$\frac{a}{b}=\frac{P}{W_2} \quad . \quad . \quad . \quad (2)$$

Multiplying (1) and (2) we find

$$\frac{a}{b}=\sqrt{\frac{W_1}{W_2}}$$

and dividing (1) by (2) $P = \sqrt{W_1 . W_2}$.

Hence, even if the balance has unequal arms, the true weight may be found by taking the geometrical mean of the apparent weights, found when the substance is placed first in one pan, then in the other.

In the case of an ordinary balance, the arms are as nearly as possible of equal lengths, and the weights W_1 and W_2 consequently so nearly alike, that instead of the geometrical we may take the arithmetical mean $(W_1 + W_2)/2$, in accordance with what has been said on approximations on page 17.

Exercise

To find the Ratio of the Arms of a Balance, and the True Weight of a Body

Proceed as follows :—

1. Place the brass cylinder provided in the left-hand pan, and find to the nearest centigram the weight required in the right-hand pan to produce a balance.

2. Place the cylinder in the right-hand pan, the weights in the left, and repeat the observation. Time will in the end be saved if between the two weighings all the weights are carefully put back into their proper places in the box.

Record as follows:—

Balance used, No. 5.
Box of weights, No. 5.
Brass cylinder, No. 7.

Apparent weight of cylinder when in left pan $(W_1) = 100{\cdot}27$ grams.
" " " " right " $(W_2) = 100{\cdot}20$ "

$$\therefore \text{ true weight } P = \sqrt{W_1 W_2} = 100{\cdot}24 \text{ grams.}$$

$$\text{and } \frac{a}{b} = \sqrt{\frac{W_1}{W_2}} = \sqrt{1{\cdot}0007} = 1{\cdot}0004.$$

SECTION XI

The Balance. II

Apparatus required.—Balance, stool, box of weights, brass cylinder, block of wood, sinker, can, and salt solution.

Exercise I

To find the Specific Gravity and the Density of a Solid unacted on by Water.

The specific gravity of a substance which is not attacked by water, is found from the apparent weight w_1 of a piece of the substance in air, and its apparent weight w_2 when suspended in water. The apparent loss of weight $w_1 - w_2$ of a solid when suspended in a liquid, is due to the upward pressure, which is equal to the weight of a volume of the liquid equal to that of the solid.

Since

$$\text{Specific gravity} = \frac{\text{Weight of body}}{\text{Weight of equal volume of water}}$$

and both numerator and denominator of the fraction may be determined by experiment, the specific gravity may be calculated.

As the specific gravity depends on a *ratio* of two weights, it is not necessary that the balance should have equal arms when used for the determination, provided the substance is always placed *on the same side* of the balance; as however the determination of *density* requires a knowledge of the true weight, the balance should have the arms equal.

Proceed as follows:—

1. Weigh the given solid in air, placing it for convenience of working in the left-hand pan.

2. Place a low wooden stool over the left-hand pan, and suspend the solid by a thin thread from the ring at the top of the pan, in a can placed on the stool. Fill the can with water, suspend the solid in the water so that it hangs clear of the sides of the can, and find the apparent weight. The weight of the silk may be neglected.

Record and calculate the specific gravity as follows:—

Specific gravity of brass cylinder, No. 9.
Balance used, No. 5.
Box of weights, No. 5.

Weight of cylinder in air w_1 . .	103·25 grms.
,, ,, water w_2 .	91·12 ,,
Loss of weight $w_1 - w_2$ . . .	12·13 ,,

Specific gravity of brass cylinder, No. 9 $\left.\right\} \frac{103 \cdot 25}{12 \cdot 13} = 8 \cdot 51$.

Repeat the observations in the reverse order. Take a mean of the results if they agree closely. If not do a third experiment, and take the mean of the three results.

The *density* of a substance being the mass per unit volume, is in the C.G.S. system of units numerically equal to the specific gravity, since the mass of 1 c.c. of water is 1 gram.

Calculate the density of brass from the volume and weight of the cylinder, and compare the density so obtained with the value of the specific gravity found by the preceding experiment.

For this purpose the diameter and length of the cylinder must be measured twice with sliding calipers and the means taken. The area of the circular base is equal to πr^2, where r is the radius of the circle. The

volume is equal to the product of the area of the base and the length.

Record as follows :—

Brass cylinder No. 9.

Mean diameter	=	1·59 cms.
„ radius .	=	·795 „
Area of base = 3·14 × ·795 × ·795	=	1·984 sq. cms.
Length of cylinder	=	6·18 cms.
Volume = 1·98 × 6·18	=	12·24 c.c.
Weight . . .	=	103·25 grms.
Density of Cylinder No. 9 = $\frac{103 \cdot 25}{12 \cdot 24}$	=	8·43

Since in the measurement of the diameter an error of half a per cent. might easily have been made, it would be useless to give the result to more than three figures. Hence, also, it would be useless to take a more accurate value for π than 3·14. The intermediate calculations should be carried on by shortened multiplication to four figures, in order to insure the accuracy of the third in the final result.

Note that by reversing the above calculations, the diameter of a cylinder, the length and density of which are known, can be found by weighing the cylinder.

EXERCISE II

To find the Specific Gravity of a Liquid

Find the specific gravity of the given liquid by weighing the brass cylinder in the liquid, taking account of the weights in air and in water previously determined.

If w_1 = weight of solid in air
w_2 = „ „ water
w_3 = „ „ liquid
then $w_1 - w_2$ = loss of weight in water
= weight of an equal volume of water
and $w_1 - w_3$ = loss of weight in liquid
= weight of an equal volume of liquid

$$\therefore \text{ specific gravity of liquid} = \frac{w_1 - w_3}{w_1 - w_2}$$

Record as follows :—

Weight of cylinder in air	= 103·25 grms.		
,, ,, in water	= 91·12 ,,	∴ Loss = 12·13.	
,, ,, in liquid	= 90·40 ,,	∴ Loss = 12·85.	

$$\text{Specific gravity of liquid} = \frac{12 \cdot 85}{12 \cdot 13} = 1 \cdot 059.$$

Repeat the weighings in the reverse order and take a mean of the results obtained in the two cases.

EXERCISE III

To find the Specific Gravity of a Solid lighter than Water

To the hook or ring at the top of the left-hand pan of the balance, attach, by means of a piece of thread about 5 cms. long, the "sinker" provided. See that when a can of water is placed on the stool underneath, the top of the sinker is about 1 cm. below the surface of the water when the pointer of the balance is at zero. Place weights W_1 in the other pan, till the balance is in equilibrium. Now place the body, the specific gravity of which is required, in the raised pan, and again balance by weights W_2 in the other pan. $W_2 - W_1$ is then the weight of the body. Now place the body underneath the sinker in the water, and balance again by weights W_3. $W_2 - W_3$ is then the apparent loss of weight in water. The specific gravity of the body is therefore

$$\frac{W_2 - W_1}{W_2 - W_3}.$$

Record as follows :—

Specific gravity of wood.

Weight to balance sinker in water	$W_1 = 67 \cdot 20$ grms.
,, ,, ,, ,, and body in air	$W_2 = 92 \cdot 91$,,
sinker and body in water	$W_3 = 45 \cdot 90$,,

$$\text{Specific gravity of block No. 9} = \frac{92 \cdot 91 - 67 \cdot 20}{92 \cdot 91 - 45 \cdot 90} = \frac{25 \cdot 71}{47 \cdot 01} = \cdot 547.$$

Repeat the observations in the reverse order and take a mean of the results obtained.

SECTION XII

The Barometer

Apparatus required.—Fortin barometer.

In some physical experiments as, *e.g.*, the determination of the boiling point of water, the magnitude of the atmospheric pressure affects the result, and it is therefore necessary to measure it. This is done by means of a *barometer*, Fig. 18, where AB is a bent tube open at one end, containing mercury, which in one branch of the tube reaches up to a level H, in the other up to the level K; the space between H and A being a vacuum. If a horizontal line be drawn through K, so as to cut the mercury in the other branch of the tube at K′, then from hydrostatic principles the pressure at K must, since there is equilibrium, be the same as at K′. The pressure at K is the atmospheric pressure, the pressure at K′ is that due to a column of mercury of a height equal to the vertical distance between H and K′. Hence the height of this column will serve as a measure of the atmospheric pressure. It follows that what we want to measure in a barometer is *the vertical distance between the free surfaces of the mercury in the two limbs of the barometer.*

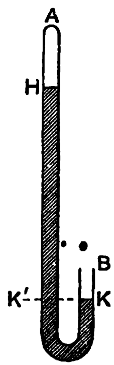


FIG. 18.

It remains to discuss the practical question how this vertical distance is to be measured.

Note in the first place, that as the mercury alters its level in one limb, it must necessarily do so in the other. If the cross section of the tube is the same in both limbs, and the mercury falls a centimetre at H, it must necessarily rise by the same amount at K. Hence the total diminution in the difference of level is two centimetres. If the cross section of the tube at K is greater than at H, then for a fall of a centimetre at H, the rise at K will be less than a centimetre; and in general the simultaneous changes of level will be inversely as the areas of the free surfaces. If, as in Fig. 19, the barometer tube is placed in a trough of mercury, the level in the trough will alter very little. Hence, to find the height of a barometer accurately, we must either measure directly the difference in level, or observe the upper surface and know the ratio of the cross section of the tube to that of the trough. The first plan is the more accurate one, and is always adopted in barometers which are used for scientific purposes.

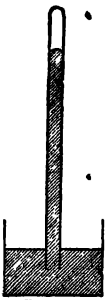


FIG. 19.

In the most common form of barometer (Fortin's) a scale is attached to the tube in such a way that an ivory pointer marks the level of the zero division. If this ivory pointer is exactly in contact with the upper surface of the mercury in the trough, the scale reading, taken in the manner indicated further on, should give the height of the barometer.

EXERCISE

To read the Barometer accurately

To set the mercury to the ivory pointer.—The trough of mercury is closed at its lower end by a flexible leathern bag,

the bottom of which can be raised or lowered by means of a screw fixed into the base of the barometer tube. If this screw is turned, the surface of the mercury in the trough moves, and may be brought just into contact with the pointer. This can be done to a high degree of accuracy if the reflection of the pointer in the mercury is watched. As the mercury is gradually raised, the pointer and its image approach, and the adjustment is complete when they appear just to come into contact.

To read the level of the mercury in the tube.—A Vernier is attached to a movable tube, the bottom end of which should be first raised clearly above the convex surface of the mercury, and then carefully lowered until it appears to be just in contact with the mercury. Parallax is avoided by keeping the eye always at such a level that the back lower edge of the Vernier tube appears to coincide with the front lower edge (Fig. 20). When the setting is made,

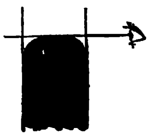


FIG. 20.

move the eye up and down to be sure that in any position of the eye no line of light appears between the central part of the surface of the mercury and the edge of the tube. Owing to the convexity of the surface there will be some light at the sides. The Vernier is now read in the manner described in the exercise "Vernier."

As the scale is set vertically once for all, the reading of the scale and Vernier gives directly the height of the barometer, but before the atmospheric pressure can be deduced from it, some important corrections are necessary.

Correction due to the temperature of the mercury and scale.—It is generally agreed to "reduce" the height of the barometer to that which would be observed, if the scale and mercury were at a temperature of 0°C., and tables have been constructed which give the correction to be applied to the barometric height observed at any temperature, in order to obtain the "reduced" height at 0°C. This correction amounts to ·186 cm. if the barometer stands at 76 cm. and the temperature is 15°, provided the scale is made of brass or of some metal having about the same coefficient of expansion. If no tables are available, a formula of reduction may be easily found. For, if a is the *apparent* coefficient of linear expansion of the mercury with respect to the material of the scale, the height h_0 of the barometer at 0° will become h at $t°$ where

$$h = h_0(1 + at).$$

The correction to the observed height, h, is therefore $-ah_0t$. If the scale is made of brass, a may be taken as 0·000163. When t is higher than 0° C., the correction has a negative sign, because the observed height h is greater than the reduced height h_0. In most observations it will be sufficient to know the barometric height to 0·1 mm., and h may be substituted for h_0 in the correction, which may then (see pp. 15, 16) be put into the form

$$-[\cdot 186 + 0\cdot 0024(h - 76) + 0\cdot 0124(t - 15°)],$$

where h represents the observed height in centimetres, and the correction is also given in centimetres. The second and third terms will be small, and are easily calculated to the required degree of accuracy. The expression will introduce errors less than ·005 cm., so long as the product $(h-76)$ $(t-15)$ is less than 30, that is if, for instance, the barometer stands between 73 and 79 cm., and the temperature lies between 5° and 25°, which is nearly always the case near the sea level.

The temperature of the mercury should be read on the thermometer fixed to the barometer, care being taken that

the presence of the observer while adjusting the barometer, does not introduce an error by raising the temperature of the air in the neighbourhood.

Correction for gravitational constant.—The barometric heights observed at different places are not strictly proportional to the atmospheric pressures, as they depend on the value of gravity, which differs at different places. Thus, for the same atmospheric pressure, the barometer would stand nearly 4 mm. higher at the equator than at the pole, because the value of gravity is about ·5 per cent. smaller at the former than at the latter place. The height above the sea level also affects the value of gravity. In order to make the barometer readings taken at different places comparable, they are reduced to what they would be, if the constant of gravitation at the place of observation had the same value as at the sea level in the latitude of 45°. The correction in London amounts to about +·044 cm., in Manchester to +·057 cm.

Correction due to capillarity, etc.—As the diameter of the tube is smaller than that of the trough, there will be some error introduced by the convexity of the mercury surface, the pressure just within the free surface being greater than the atmospheric pressure. The exact amount of the correction to be applied depends on various circumstances. Barometers which are intended for very accurate work have tubes which are so wide that the error introduced by capillarity is always small. Wherever possible, the barometer should be compared with a standard in which the capillarity correction is eliminated. Errors of the scale divisions are often more serious than those due to capillarity, and must be specially determined.

The barometer referred to below has been compared with the standard at Kew, and found to require a correction of −·010 cm. on the metre scale, and of

−0·001 inch on the English scale. These corrections include that due to capillarity and that due to errors of the scale.

Enter and reduce the observations as follows :—

			Cm.
Barometer reading observed			75·235
Attached thermometer reading. . . .	17°·3C.		
	Cm.		
Correction due to latitude (Manchester) .	+ ·057		
		Cm.	
Capillarity and scale correction . .		− ·010	
Correction due to temperature—(1) − ·186			
(2) $+0·0024 \times ·765 =$ + ·002			
(3) $-·0124 \times 2 3 =$ − ·029		− ·213*	
Sum of corrections	+ ·057	− ·223 =	− ·166
Reduced barometer			75·07

* The temperature correction may be calculated directly from the expression $-aht$, p. 62, thus :—

$$-·000163 \times 75·2 \times 17·3 = -·213 \text{ mms.}$$

The final value is only given to two decimal places, as the corrections are not intended to be more accurate.

SECTION XIII

Elasticity

Apparatus required.—Rubber band with wooden centimetre scale and two glass millimetre scales, weights, calipers.

When a body is changed in volume or shape, it is said to be strained, the deformation being called a *strain.* Strains are generally produced by forces applied to the surfaces of bodies; the inside portions of the body in that case, are kept in a state of strain by internal forces acting between adjacent parts of the body. These internal forces are called *stresses.* A stress at any point is measured by the resultant force per unit surface. A unit force distributed uniformly over a unit surface would therefore be a unit stress.

The relation between the stresses and the strains they produce is expressed by Hooke's Law, which states that "*The stress is proportional to the strain it produces.*" This law holds for small stresses only, but is sensibly correct for deformations within the limits of elasticity, that is to say, for stresses which are not sufficiently great to produce a *permanent* deformation of the body.

If a wire suspended vertically is stretched by weights, the strain is measured by the elongation per unit length, and the stress by the force applied per unit cross section. If the wire, originally of length L, were stretched until its length became $L+l$, the elongation would be measured

by $\frac{L}{l}$. If the force applied were P and the area of cross section a, the stress would be $\frac{P}{a}$.

As the strain is measured by a ratio of lengths, the numerical measure of the *strain* will be independent of the unit of length. This is not the case with the numerical measure of the *stress*. The unit of force varies directly as the unit of length adopted, while the unit of surface varies as the square of the unit of length; hence the unit of stress will vary inversely as the unit of length.

According to Hooke's law $\frac{P}{a}$ is proportional to $\frac{l}{L}$ or

$$\frac{P}{a} = M\frac{l}{L}$$

where M is a constant which is called Young's Modulus. The equation assumes that l is small compared to L.

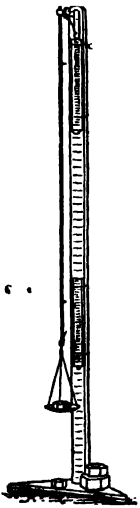


FIG. 21.

The object of the present exercise is to illustrate the method by means of which Hooke's law may be verified for the longitudinal stretching of wires. As however in the case of a metal wire, the stretching is so small that a microscope would be required for its measurement, an indiarubber band which stretches considerably is taken. Hooke's law under these circumstances cannot be expected to apply strictly, but the exercise will serve to show its general nature.

An indiarubber band about 50 cms. long which may be attached to a suitable stand is provided (Fig. 21). A scale pan is attached to the lower end of the band, and two pins are thrust through the indiarubber so that the points protrude a little on one side, the other ends of the pins being cut off. The distance between the points of the two pins, which for

convenience may be adjusted to about 40 cms., is the length to be measured while the band is being stretched by different weights placed in the pans. In order that this distance may be accurately determined, two millimetre scales etched on the front of mirror glass are fixed to the stand, so that one scale is behind each of the pins. A strip of silvering is removed from each mirror in order that the centimetre divisions on the stand may be seen. The centimetre divisions on each scale should be made to coincide with the centimetre divisions on the stand.

The positions of the pins are read on the scales, parallax being avoided by means of the mirror. It is most convenient for the purpose if the scales are placed so that the pins move just along the edges of the divisions.

The load in the pan should be gradually increased to 250 grams by the repeated addition of 50 grams, and then diminished to zero in the same way. The results should be entered as follows:—

Rubber band No. 3. Diameter = ·42 cm.

Weight.	Reading of upper pin.	Reading of lower pin.	Distance between pins.	Δ
pan only	12·65	55·84	43·19	1·75
,, + 50 grms.	12·94	57·88	44·94	1·76
,, + 100 ,,	13·28	59·98	46·70	2·06
,, + 150 ,,	13·66	62·42	48·76	2·22
,, + 200 ,,	14·04	65·02	50·98	2·55
,, + 250 ,,	14·56	68·09	53·53	2·30
,, + 200 ,,	14·17	65·40	51·23	2·13
,, + 150 ,,	13·76	62·86	49·10	2·04
,, + 100 ,,	13·39	60·45	47·06	1·80
,, + 50 ,,	13·03	58·29	45·26	1·74
pan only	12·72	56·24	43·52	

Each number in the column headed Δ is the difference between the two numbers above and below it respectively

in the preceding column, and it will be seen from them that the addition of equal weights produces a greater and greater effect on the length of the string as the load increases, the first addition of 50 grams only producing an elongation of 1·75 cms., while the elongation produced by the increase of the load from 200 to 250 grams was 2·55 cms. Another fact which appears from inspection of the last column in the above table, is that for the same load the length is slightly larger as the weights are taken off, than it was as they were being added. When the weights are entirely removed the string has been permanently lengthened.

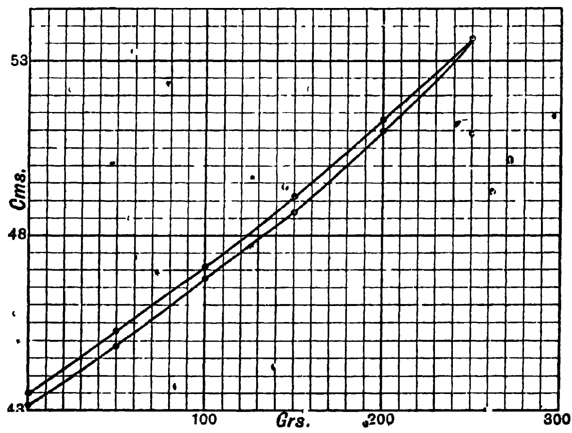


FIG. 22.

Represent the result graphically.—The result of the experiments will be rendered more apparent to the eye by plotting the observations, as described in Section III. Horizontal distances may be taken to represent the weights placed on the pan, while vertical distances indi-

cate in cms. the excess of the length of the string between the pins over a length somewhat less than the smallest length measured. If the student's note-book is divided into cm. squares, it will be found convenient to take the length of one side of a square to represent an elongation of ·5 or 1 cm. on the vertical axis, and a weight of 10 or 20 grams on the horizontal axis (Fig. 22). The lines drawn through the points observed are slightly curved, which shows that Hooke's law is not strictly true.

We may however find an approximate value for Young's modulus of elasticity for indiarubber when the load is small. If r is the radius of the string, πr^2 is the area of the cross section. An increase of weight of 50 grams produced an increase of length from 43·19 to 44·94 cms. Hence the *strain* is measured by $\frac{1\cdot75}{43\cdot19} = \cdot0403$. The corresponding *stress* in grams weight per sq. cm. was $\frac{50}{\pi r^2}$ and $2r$ was measured to be ·42 cm. Hence the stress $= \frac{50}{\pi(\cdot21)^2} = 363$ grams weight per sq. cm. $= 363 \times 981$ dynes per sq. cm., and Young's Modulus $= \frac{363}{\cdot0403} = 9000$ grams per sq. cm. $= 9000 \times 981$ dynes per sq. cm.

Measure the diameter of the indiarubber string by means of the screw gauge, calculate Young's modulus, and record as follows :—

Rubber band "A."

Length with pan + 50 grams	44·94 cms.
,, ,, only	43·19 ,,
Extension for 50 grams	1·75 ,,
∴ Strain = 1·75/43·19	= ·0403
Radius of rubber = ·42/2	= ·21 cm.
∴ Area of cross section = πr^2	= ·138 sq. cm.
Stress = 50/·138	= 363 grams per sq. cm.
∴ Young's Modulus	= 9000 ,, ,,
	≐ 9000 × 981 dynes per sq. cm.
	= 8,830,000 ,, ,,

SECTION XIV

Boyle's Law

Apparatus required.—A closed glass tube connected by rubber tubing with a similar open tube, both capable of being moved up and down a vertical graduated scale.

Definition.—The law of Robert Boyle, published in 1662, states that the *volume of a mass of gas at constant*

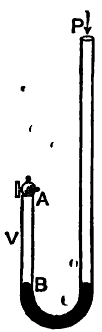


FIG. 23.

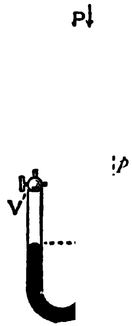


FIG. 24.

temperature varies inversely as the pressure to which it is subjected. Thus consider the air enclosed by mercury in the shorter limb AB of the bent tube, Fig. 23. If the

tap at A is opened the mercury assumes the same level in both tubes. Let now the tap be closed. A mass of air of volume which we will call V, will be isolated in AB at atmospheric pressure—*i.e.* at the pressure corresponding to the height of the barometer at the time, which is, let us say, P centimetres of mercury. If mercury be now poured down the longer limb, until the difference in the levels of the mercury in the two tubes (*see* Fig. 24) is p centimetres, the pressure to which the gas is now subjected is $P + p$, and the volume of gas in the tube AB will decrease, becoming equal, say, to V'. Boyle's Law states that :—

$$\frac{V}{V'} = \frac{P + p}{P}$$

or

$$VP = V'(P + p),$$

which signifies that *the product of the volume of a given mass of gas at constant temperature and the pressure to which it is subjected, is a constant however one of the two quantities is varied.*

Exercise

To verify the above Law

An apparatus by means of which the pressure on a gas may be varied at will is necessary. A bent tube, such as Fig. 23, would serve the purpose, but as it is difficult to keep stopcocks tight, the end A is better closed. The apparatus shown in Fig. 25 has been found convenient to illustrate the law. A is a closed glass tube connected with an open glass tube B by means of indiarubber tubing. Between the glass tubes is a vertical millimetre scale S, on which the difference of levels of the mercury columns in the tubes may be read off. A and B are attached to movable boards which can be suspended by hooks from the different holes in the stand carrying the vertical scale,

and thus raised or lowered. In this way the pressure to which the gas in the closed tube is subjected can be varied. Attached to the board carrying the closed tube is a thermometer, on which the temperature of the air is indicated, and it will be assumed that the air in the tube AB is at the same temperature. In moving the board to which A is attached, care should be taken not to heat the tube with the hand.

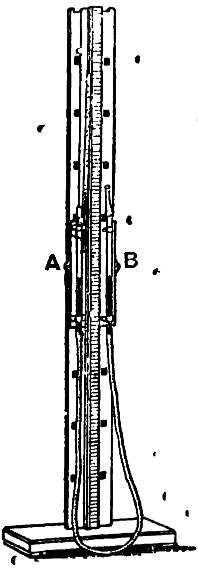


FIG. 25.

Before commencing the experiment read the barometer.

Place the tubes A and B about the middle of the stand, as shown in the figure. Read on the scale, using the T-square provided, (*a*) the position of the *inside* of the top of the closed tube A, (*b*) the level of the mercury in that tube, and (*c*) the level of the mercury in the open tube B. In each case the highest point of the convex surface should be read as indicated in Fig. 20, p. 61. The bore of the tube A may be assumed to be sufficiently uniform to enable the difference of the first two readings to be taken as proportional to the volume of air in the tube.

Read the thermometer attached to the board A.

Now move the board carrying the open tube B to the next higher position, and take readings as before. Continue raising B step by step, taking readings each time, till it is as high as it can be. To further increase the pressure to which the gas in A is subjected, lower A step by step, taking readings as before, till it is at its lowest position, then raise it to its middle position, and bring down B to the same level. Take readings again in this condition.

Now lower B so as to take readings of volume under diminished pressure, and continue the process till B gets to its lowest position. Then begin to raise A, and after having taken it as high as it is possible without the mercury being poured out of B, bring both A and B back to the middle positions, and take readings.

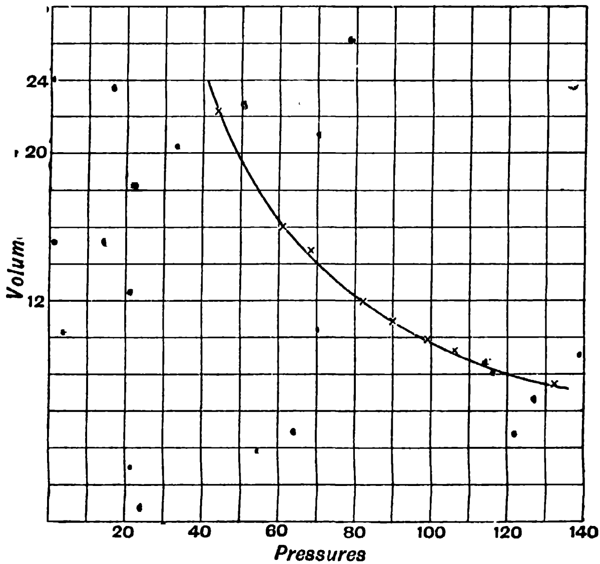
Record and calculate as follows :—

Barometer 75·26 cms. Apparatus No. 4.							
Closed tube. Top. cms.	Closed tube. Mercury. cms.	Open tube. Mercury. cms.	Temp.	Volume of Air.	Difference of pressures open —closed.	Total pressure. cms.	Product of pressure and volume.
61·89	49·91	56·97	19·6	11·98	7·06	82·32	985
61·89	51·00	65·70	19·7	10·89	14·70	89·96	980
61·89	51·95	74·55	19·8	9·94	22·63	97·86	973
61·89	52·65	83·55	19·9	9·24	30·90	106·16	980
51·90							
41·89							
31·91	24·55	81·16	20·2	7·36	56·61	131·87	972
41·90							
51·90							
61·89	&c.						
61·89	49·92	56·98	20·9	11·97	7·06	82·32	984
61·89	47·43	40·25	20·9	14·46	−7·18	68·08	983
61·89	45·90	32·10	21·0	15·99	−13·80	61·46	983
81·87	59·49	28·57		22·38	−30·98	44·28	989
	&c.						
61·89	49·91	56·97	21·2	11·98	7·06	82·32	985

It is seen that the errors of observation render the result doubtful to about 1 per cent., which is probably due to the difficulty of measuring the volume correctly.

Taking distances along a line from left to right in your note-book to indicate pressures, and along a line up and

down to represent volumes, draw a curve connecting the observed volumes of the gas with the pressures, as follows :—



The curve thus obtained is a rectangular hyperbola.

PART III

HEAT

SECTION XV

Determination of the Freezing and Boiling Points on a Thermometer

Apparatus required.—Two thermometers, a freezing point-can, and a boiling point tube.

The object of these exercises is to test the indications of two thermometers at the temperatures of melting ice and of boiling water.

EXERCISE I

Determination of the Freezing Point

Two thermometers, one with a Centigrade, the other with a Fahrenheit scale, and a canister, perforated at the bottom (Fig. 26), are provided. This vessel is to be filled with small pieces of ice, the smaller the better, and a small can placed underneath so as to receive the water drained off the melting ice. The surface of the ice should be level with the top edge of the vessel. With a rod of cross section equal to that of the thermometer bulb, make a *vertical* hole in the ice underneath the spring clip fixed to the can, which is intended to hold the thermometer. The hole should be of such a length that when the bulb is inserted so as to reach the bottom of it, the scale division marking the freezing point is at the level of the top of the canister,

Now insert the thermometer carefully into the opening made. If the eye is placed in such a position (Fig. 27) that the top of the can is foreshortened into a straight line, the freezing point should just be visible. Tap the thermometer lightly with a pencil and read the indication of the thermometer when it has become quite steady, estimating to tenths of a degree. If the thermometer stands vertical, while the line of sight is horizontal, errors due to parallax are avoided.

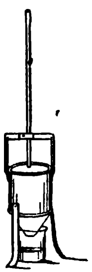


FIG. 26.

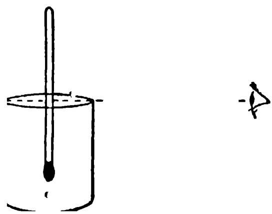


FIG. 27.

The correction of a thermometer at a given temperature is that quantity which has to be ADDED *to the reading to get the true temperature.*

Hence, if a Centigrade thermometer reads 0°·3 when placed in melting ice, which has a temperature of 0° C., the correction is − 0°·3.

Determine the correction at the freezing point for each of the thermometers provided, and record as follows :—

Fahrenheit, No. 7.

Observed freezing point . . .	31°·8
Correction at freezing point .	+0°·2

Centigrade, No. 7.

Observed freezing point . .	0°
Correction at freezing point .	± 0°·0

EXERCISE II

Determination of the Boiling Point

The determination of the error of a given thermometer at the boiling point is, for various reasons, a little more complicated than the corresponding task of finding the error at the freezing point. It is on this account that the determination of the freezing point has been carried out before that of the boiling point, although the reverse order is the one generally adopted in accurate thermometry.

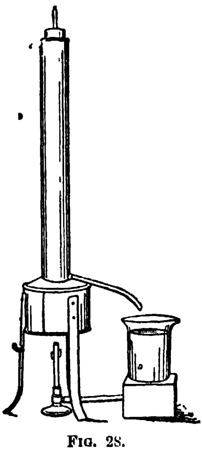


FIG. 28.

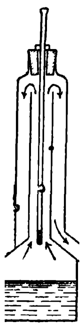


FIG. 29.

The boiling point cannot be determined by placing the thermometer in boiling water as the temperature is

affected by impurities in the water. The steam arising from the water is found, however, always to have the same temperature, *so long as the barometric pressure remains the same.*

The apparatus used in making the determination is shown in Fig. 28, with the thermometer inserted, and its inner construction will be clear from Fig. 29, where the arrows indicate the course of the steam. The steam rising from the boiling water passes through the cylindrical tube into which the thermometer is inserted, and then through an outer jacket, the object of which is to protect the inner vessel against the cooling action of the atmosphere. It is important that the pressure of the steam in contact with the thermometer bulb should be known. If the opening through which the steam escapes is moderately wide, the pressure will be sufficiently near the atmospheric pressure to be taken as equal to it; but when great accuracy is required, the apparatus is provided with a manometer to determine the pressure in the inner tube.

Before proceeding to find the error of the thermometer at the boiling-point, the temperature at which water boils under the atmospheric pressure at the time must be calculated.

On the Centigrade scale, 100° is defined as the temperature of the steam arising from water boiling under the barometric pressure of 76 cms. of mercury, at the sea level in the latitude of 45°, the mercury being at the temperature of freezing water.

On the Fahrenheit scale, 212° is defined to be the temperature of the steam arising from water boiling under a barometric pressure of 29·905 inches at the sea level in the latitude of London, the mercury being at the freezing point.

The two pressures, one indicated by a barometric height of 76 cms. at latitude 45°, the other by a height

of 29·905 inches at Greenwich, are identical, because although 76 cms. is equal to 29·922 inches, the gravitational constant is not quite the same in the two places owing to the difference in latitude.

If the barometric pressure is known, and differs from the normal, the boiling point may be calculated from the experimental fact that a difference of 1 cm. pressure of mercury produces approximately a difference of $0^{\circ}\cdot37$ C. or $0^{\circ}\cdot66$ F. in the boiling point; the boiling point being raised by an increase, and lowered by a decrease of pressure.

This rule will give the boiling point correct to $0^{\circ}\cdot02$ C., for pressures varying between 73 and 80 cms. If the barometer stands below 73 cms.—as, for instance, when the place of observation is considerably above the sea level—tables which give the relation between the boiling point and the barometric pressure should be consulted.

The boiling points on the two thermometers should now be found as follows :—

(1) Insert carefully one of the thermometers into the apparatus (Fig. 28), so that the nominal boiling point is about one or two divisions above the short cork slipped on to the thermometer to support it, and wait till the water boils.

(2) While the thermometer is taking up its final temperature, read the barometer according to the instructions given in Section XII., applying the corrections.

(3) Watch the thermometer for two or three minutes after it has apparently stopped rising. If the thread does not appear above the cork, the thermometer must be slightly raised. When its indication has become steady, push the thermometer down till the end of the thread is only just in sight; then read the temperature indicated, estimating to $\cdot1^{\circ}$.

(4) Make corresponding observations with the other thermometer, reading the barometer once more to check the previous result.

The results are entered and reduced as follows :—

Normal height of Barometer		76·00 cms
Observed Height	75 24 cms.	
Corrections	– ·17 ,,	
Observed height corrected		75·07 ,,
Difference		0·93 cm.

Corresponding *Depression* of Boiling Point, Fahr. = ·66 × ·93 = 0°·6
,, ,, ,, Cent. = ·37 × ·93 = 0°·3

Fahrenheit, No. 7.

	Freezing point.	Boiling point at 75·07 cms. pressure.
True value	32°	211°·4
Observed value . . .	31°·8	212°·4
Corrections	+ 0°·2	– 1°·0

Centigrade, No. 7.

	Freezing point.	Boiling point at 75 07 cms. pressure.
True value .	0°	99°·6
Observed value	0°·0	99°·2
Corrections . .	± 0°·0	+ 0°·4

SECTION XVI

Correction and Comparison of Thermometers

Apparatus required.—Two thermometers, the errors of which at the freezing and boiling points are known, and a vessel for heating water.

In this exercise the indications of the two thermometers, for which the corrections at the freezing and boiling points have been previously determined, are to be corrected and compared at intermediate temperatures.

Take one of the brass cans supported on legs which raise it to a suitable height for heating over a Bunsen burner, and fill it with tap water, having a temperature probably lower than that of the room. Hold the two thermometers which are to be compared, together in the left hand, with their bulbs immersed, and with the help of a stirrer held in the right hand, or by using the thermometers themselves as a stirrer, bring the water to a uniform temperature. Read off the indicated temperatures.

Heat the water by means of a burner placed under the can, until the temperature is about 20° C. Remove or turn down the burner, and after thoroughly stirring the water, read the thermometers. Repeat the observations at temperatures about 30°, 40°, and 50° C. At the higher temperatures, the burner should be turned down and left under the can, in order to keep the temperature of the water constant for a minute before the thermometers are read. The stirrer should be used continuously but not violently.

Convert the Fahrenheit readings into readings on the Centigrade scale, and tabulate as follows :—

Fahrenheit No. 7.	Centigrade No. 7.	Fahr. converted to Cent.	Difference C – F.
31°·8	0°·0	– 0°·1	+ 0°·1
60 ·2	15 ·6	+15 ·7	– ·1
73 ·7	22 ·9	23 ·2	– ·3
88 ·3	30 ·9	31 ·3	– ·4
102 ·1	38 ·5	38 ·9	·4
121 ·6	49 ·3	49 ·7	·4
212 ·4	99 ·2	100 ·2	– 1 ·0

The errors to which a thermometer is liable may be classified under three heads :—

(1) Errors in the freezing and boiling points.

(2) Errors due to the inequality of the bore of the thermometer tube.

(3) Errors of graduation.

The magnitudes of the errors at the boiling and freezing points, are easily determined, as in the previous exercise.

Errors due to inequality of the bore may be corrected by a proper graduation of the tube. If the bore of the thermometer tube had the same width throughout, the maker would simply have to divide the distance between the freezing and normal boiling points into 100 (or 180) parts, to get a correct scale of temperature ; but wherever the bore is narrower, the degrees should be further apart, if an equal number of degrees is to correspond everywhere to an equal apparent increase of volume of the mercury. In thermometers intended to read correctly to a small fraction of a degree, the stem, before being graduated, is "calibrated" ; that is to say, the widths of the bore at different points are compared by measuring the length of the same thread of mercury at those points. But even then errors of graduation may be present, through the divisions not being placed exactly where they should be.

In the thermometers provided, the divisions are equal in length, and errors due to inequality of the bore may therefore exist. It is required to find how much of the observed difference between the Centigrade and Fahrenheit thermometers, is due to the errors at the freezing and boiling points, which are known, and how much to inequalities in the bores of the tubes.

We correct, in the first place, the readings of the thermometers for the known errors at the freezing and boiling points, by the following graphical method :—

Turn the note-book in which results are recorded, so that the longer side of the page goes from left to right. Let distances from left to right represent degrees on the thermometer, and distances up and down the corrections to be applied to the readings. As the corrections may be either positive or negative, draw the axis of temperature half way up the page. Next fix the scales. If the note-book contains twenty lines parallel to its shorter side, it will be possible to represent the whole range from freezing to boiling water on a Fahrenheit scale if each division is taken to equal 10°. Hence if the left-hand corner of a square touching the axis of temperature is taken to correspond to 32°, the right-hand corner of the square in the axis of temperature will correspond to 42°, and so on. The scale of corrections is conveniently chosen so that the side of each cm. square represents 0·1°. In the example given in the preceding section, the Fahrenheit thermometer No. 7 read 31°·8 at the freezing point, and 212°·4 when it should have read 211°·4. Take a point therefore in the horizontal axis corresponding to 31°·8 (this will be a little to the left of the vertical axis), and pass vertically upwards to the second division, which represents the correction +0°·2. Similarly at the point corresponding to 212°·4, go down vertically to the tenth line, which represents an error of −1°·0. Join the two points by a straight line,

the correction at any reading is then equal to the vertical distance from the point representing that reading,

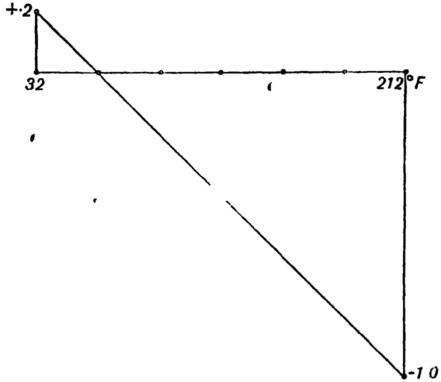


FIG. 29a.

to the joining line. The point of intersection of the straight line with the horizontal axis, gives the temperature (about 62°), at which the thermometer reads correctly.

A similar line should now be drawn for the Centigrade thermometer. Here the origin may conveniently be called 0°, and the horizontal side of each square may represent 5°C.

Determine in this way all the corrections to the original readings, and construct a table as follows:—

FAHRENHEIT, No. 7.			CENTIGRADE, No. 7.			Corrected F. reduced to C.	Difference C − F.
Observed.	Correction.	Corrected.	Observed,	Correction.	Corrected.		
31°·8	+0°·2	32°·0	0°	0°	0°	0°	±0°·0
60·2	− ·0	60·2	15·6	+ ·1	15·7	15·7	+ ·0
73·7	− ·1	73·6	22·9	+ ·1	23·0	23·1	− ·1
88·3	− ·2	88·1	30·9	+ ·1	31·0	31·2	− ·2
102·1	− ·3	101·8	38·5	+ ·2	38·7	38·8	− ·1
121·6	− ·4	121·2	49·3	+ ·2	49·5	49·6	− ·1
212·4	− 1·0	211·4	99·2	+ ·4	99·6	99·6	±0·0

The difference between the two thermometers is, as the table shows, reduced almost to the errors of observation, and the principal source of the difference between the uncorrected readings of the two, is therefore to be found in the somewhat large error of the Fahrenheit thermometer at the boiling point. So far as these observations go, they do not point to any serious errors of calibration.

When a thermometer is raised to a high temperature, and then quickly cooled, the bulb does not contract at once to its original volume, but requires a considerable time, towards the end of which the diminution is taking place very slowly. In consequence of this, thermometers which have been heated considerably in the process of manufacture, often show a gradual rise of their freezing points for years afterwards. It has been found that thermometers which after being raised to a high temperature are cooled very slowly, do not show this alteration in their freezing points and the best thermometers are now treated in this manner. Such thermometers, however, show still a temporary lowering of the freezing point after being heated to the temperature of boiling water. The extent of this lowering depends on the nature of the glass, and must be determined if temperatures are to be measured correctly to less than a tenth of a degree.

Clinical thermometers, which are still often made of a kind of glass which produces a large variation of the zero point, should be tested from time to time, if they are required to indicate the temperature accurately.

EXAMPLE.—A clinical thermometer has been tested and found to have a correction of $+0^\circ{\cdot}3$ at 95° F. and $-0^\circ{\cdot}2$ at 105° F. Assuming that the bore of the tube is uniform, what is the correction at 98° F.?

SECTION XVII

Specific Heat. I

The Water Calorimeter

Apparatus required.—Calorimeter, two Centigrade thermometers, and a small flask.

An apparatus used for measuring quantities of heat is called a calorimeter. The calorimeter used in the present exercise, consists of a small vessel made of sheet copper, supported on corks and placed inside a larger vessel (Fig. 30), which protects it against irregular changes of temperature due to air currents, and to some extent from loss of heat due to radiation and conduction to the air. Quantities of heat are measured by the increase of temperature of a known mass of water, placed inside the calorimeter, the amount of heat necessary to raise 1 gram of water from 9°·5 to 10°·5 C. being taken as the unit (called a gram-degree).

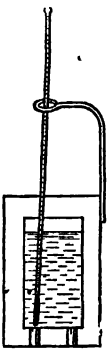


FIG. 30

As the successful handling of a calorimeter requires some skill, we begin with the simple exercise of mixing known quantities of hot and cold water, and determining the temperature of the mixture. This will enable us to verify that the amount of heat necessary to raise the temperature of 1 gram of water 1°C. at any temperature is with sufficient accuracy for our purpose equal to 1 gram-degree defined as above.

EXERCISE

If a quantity M of warm water at a temperature T, be mixed with a quantity m of colder water at a temperature t, and if θ be the temperature of the mixture, the mass M of warm water has lost a quantity of heat which, if we assume that the heat required to raise one gram of water one degree is the same at all temperatures, is expressed in gram-degrees by $M(T-\theta)$, while the cold water has gained an amount of heat $m(\theta-t)$, and if there has been no other gain or loss of heat, these quantities must be equal. Hence

$$M(T - \theta) = m(\theta - t) \quad . \quad . \quad . \quad . \quad (1)$$

from which equation θ can be calculated.

It has been assumed that the quantity of heat required to raise a certain mass of water through one degree, is the same whatever the temperature, and this is not strictly true; but to the degree of accuracy attainable in ordinary calorimetrical experiments, the error thus introduced is inappreciable.

It is required to find how far the value θ determined from equation (1), agrees with that found by observation. The experiment is carried out as follows:—

1. Two Centigrade thermometers are used, one to observe the temperature of the hot water, the other that of the cold water in the calorimeter, and it is necessary to determine whether there is any difference of the indications of the two thermometers when at the same temperature. The hot water in the experiment will be heated to about 50° C., and the thermometers should be compared at that temperature. Fill with warm water one of the vessels used in the comparison of thermometers, adjust the

temperature to about 50° C., place both thermometers inside, stir the water well, and read the temperatures indicated.

Note the difference thus :—

Thermometer No. 47	. . .	52°·4
" " 24	. . .	52°·3
Difference	No. 47—No. 24 =	0°·1

2. Weigh the calorimeter empty. Place an additional 50 grams on the balance pan, and pour into the calorimeter sufficient water to again produce a balance. If too much has been poured into the calorimeter, add weights till a balance is produced. The difference between the total weight and that of the calorimeter alone, is the weight of water added.

Place one of the thermometers in the water in the calorimeter.

3. In the same way weigh out 50 grams of water in a two-ounce flask, introduce the second thermometer, and heat slowly on a retort stand covered with wire gauze, over a small Bunsen burner.

4. While the water in the flask is heating, stir the water in the calorimeter, and see that its temperature is steady.

5. When the warm water is at about 50° C., remove the flame, stir carefully but thoroughly with the thermometer, watching the latter all the time.

6. Read and note the temperatures of the water in the calorimeter, and of the warm water in the flask, estimating to $\frac{1}{10}$th degree, then take the thermometer out

of the flask, shake the water adhering to it back into the flask, and pour the water quickly into the calorimeter.

7. Stir the water in the calorimeter and watch the thermometer closely as it rises; read to $\frac{1}{10}$th degree the highest temperature it reaches.

8. There will be some water left in the flask; the amount which has been poured out should be found by again weighing the flask and contents, and subtracting the weight found from the total weight.

The success of the experiment will depend on the quickness with which it is performed; and a first trial will probably not give a satisfactory result. In that case repeat the experiment, but work out *all* the results, whether they seem satisfactory or not, making a note, however, of the reason why a particular experiment is not considered trustworthy. As the hot water should be poured into the calorimeter directly the thermometer in the flask is read and taken out, no time is left to put the reading down on paper until the final temperature has also been read off. As the experiment will be useless if the temperature of the warm water is not correctly recorded, proceed as follows. First read and *record* the whole number of degrees indicated by the thermometer in the hot water, leaving the decimal place open. The temperature will only vary a few tenths of a degree while this is done. Then read the decimal of a degree and empty the flask. The decimal is easily remembered until time is found to write it down.

Calculate by equation (1) what the temperature θ of the mixture ought to be, and record as in the first column below:—

Calorimeter used No. 6.

Thermometer in warm water, No. 47.

,, ,, calorimeter, No 24.

Mass of flask and warm water.	75 0	75·5	grams.
,, ,, left in it	25·5	26·1	
,, warm water used (M) .	49·5	49·4	

Temperature of warm water observed on No. 47	52°·3	55·0	C.
Temperature of warm water reduced to No. 24 (T)	52°·2	54·9	
Mass of calorimeter and cold water.	71 ·1	98·9	grams.
,, ,, ,, alone	21 ·1	21·0	
Mass of water in calorimeter (m)	50 ·0	77·9	
Temperature of water in calorimeter (t)	17°·2	15·5	C.
,, ,, mixture observed	33°·9	30·2	
Temperature of mixture calculated without water equivalent of calorimeter (θ)	34°·6	30·8	
Temperature of mixture calculated with water equivalent of calorimeter (θ')	34 ·2	30·4	
Temperature calculated — Temperature observed	·3	·2	

The difference between the observed and calculated values is due to three causes, all tending to make the observed temperature smaller than the calculated one. In the first place, the warm water is cooled by contact with the cooler neck of the flask and with the air while it is being poured into the calorimeter. Secondly, the warm water heats not only the water in the calorimeter, but also the calorimeter itself, the stirrer, and the thermometer in the calorimeter. Thirdly, the calorimeter loses part of its heat by radiation and conduction to the air.

We may easily determine the magnitude of the effect produced by the second cause. If w is the mass of the calorimeter and c its specific heat, then wc is its thermal capacity, or, as it is called, its "water equivalent," *i.e.* the mass of water which would require the same amount of heat to raise its temperature 1°C. that the calorimeter does. The specific heat of copper is nearly ·1; the weight of the calorimeter and stirrer in the above example was 21·1 grms., neglecting the weight of the cork feet; hence the water equivalent is 2·1 grms.; the water equivalent of the thermometer is approximately ·5 grms. The thermal capacity of the calorimeter and accessories is therefore 2·6 grms. This should be added to the mass of water in the calorimeter, in order to get the full capacity 52·6 (m) of the matter heated, and the calculation

of θ repeated. When this is done the calculated value of θ'' becomes $34^\circ{\cdot}2$ C.

Perform another experiment, using about 75 grams of water in the calorimeter, and tabulate the results of the two experiments as shown above.

It will be seen that by taking into account the water equivalents of calorimeter and thermometer, the difference of the calculated and observed temperature has been reduced to about one half its previous value.

The agreement between observed and calculated temperatures is sufficient to justify the assumption we have made in the calculation, that the quantity of heat necessary to raise a given mass of water 1°C. is the same whatever the initial temperature of the water.

SECTION XVIII

Specific Heat II—Water Equivalents

Apparatus required.—Calorimeter, two Centigrade thermometers, and a flask.

The water equivalent of a body is the mass of water which would require the same amount of heat to raise its temperature one degree that the body requires. The object of the following exercises is to show how its value may be determined experimentally.

EXERCISE I

To find the Water Equivalent of a Calorimeter

This can be done to a sufficient degree of accuracy by pouring some warm water, the temperature of which has been observed, into the empty calorimeter and noting the fall of temperature of the water. (First weigh the calorimeter and stirrer,) then place a thermometer in it. Allow the thermometer to remain for a few minutes, then observe the temperature t and remove it. Put into the flask provided, an amount of water which, when poured into the calorimeter will just cover the bulb of a thermometer placed in the calorimeter. Place a thermometer in the flask and heat to a temperature T about 35° C, then remove the flame and stir the water well. As soon as the temperature is steady, pour the water quickly, but carefully, from the flask into the calorimeter. Keep the thermometer in the flask with one hand while the water is being poured out, and then put it quickly into the calorimeter, and use it as a stirrer. Its temperature will be

seen to fall rapidly through a small range due to the calorimeter and stirrer taking up from the water an amount of heat, which is equal to $w(\theta - t)$ where w is the water equivalent of the calorimeter and stirrer, and θ the final temperature. After the first rapid fall, the temperature will diminish owing to radiation and conduction, and in order that this cooling should be slow, and therefore easily distinguished from the first rapid fall, which alone concerns us, the temperature of the warm water should not be higher than about 35°. The amount of water poured into the calorimeter is regulated by the fact that the smaller the quantity the greater the fall of temperature, and therefore the more accurate the experiment; on the other hand, the quantity of water when in the calorimeter must be sufficient to cover the thermometer bulb completely. Also the parts of the calorimeter not in direct contact with the water should be heated by conduction to the temperature of the water; and, as they are at the same time cooled by contact with the air, errors will be introduced if a very large portion of the calorimeter is not in contact with the water. With the apparatus provided, it will be found that a quantity of water filling the calorimeter to about one third its full contents will give sufficiently accurate results.

At the end of the experiment, weigh the calorimeter with the water it contains. Since the weight of the calorimeter has been previously determined, the mass of water (M) is thus ascertained. The heat given up by this mass is $M(T - \theta)$, hence, neglecting the small amount of heat given up or absorbed by the thermometer,

$$w(\theta - t) = M(T - \theta),$$

or the water equivalent $w = M(T - \theta)/(\theta - t)$.

Repeat the experiment and record as follows:—

Calorimeter used No. 6; Thermometer No. 24.

Temperature of calorimeter (t)	17°·2	18·3	C.
,, ,, warm water (T)	34°·8	33·1	

Temperature of water after introduction into calorimeter (θ)	34°·2	32·5	
Mass of calorimeter, stirrer and water	76 ·5	72·4	grams.
,, calorimeter and stirrer	19 ·7	19·7	,,
,, water used (M)	56 ·8	52·7	,,
Water equivalent w of calorimeter 6 by experiment	2 ·0	2·1	,,
Water equivalent w of calorimeter by weighing	1 ·97	1·97	,,

The value deduced from the observations, may be checked by calculating the water equivalent by taking the product of the weight of the calorimeter, and the specific heat of copper, which is about ·1. This gives 1·97, a number which agrees, within the errors of experiment, with the observed value.

EXERCISE II

To find the Water Equivalent of a Thermometer

Fill the calorimeter with sufficient water to cover the bulb of a thermometer placed inside it so as just not to touch the bottom. The quantity of water should be determined by weighing the calorimeter empty, and with the water in it. Observe and record the temperature of the water. Heat the thermometer, the water equivalent of which is to be determined, in a vessel of water to about 80°C., then take it out, dry the bulb with a cloth, and immerse it in the water of the calorimeter, noting its temperature just before immersion. Observe the rise of the temperature of the water by means of the thermometer which stands in the calorimeter.

Repeat the experiment.

The student should write down the equation giving the water equivalent w, in terms of T the temperature of the heated thermometer, t the initial, θ the final temperature of the calorimeter, M the total mass of water heated, *i.e.*

the water in the calorimeter and the water equivalent of the calorimeter, and from the observations calculate the water equivalent.

Record as follows :—

Calorimeter No. 6.
Thermometer in calorimeter No. 24.
Water equivalent thermometer No. 25.

Mass of water in calorimeter	28·1	30	grams.
Water equivalent of calorimeter . . .	2·0	2·0	,,
Total water equivalent (M)	30·1	32·0	,,
Temperature of heated thermometer (T) .	78·2	79°·2	C.
Initial temperature of calorimeter (t) . .	18·4	17°·3	C.
Final temperature of calorimeter (θ) . .	19·3	18°·3	C.
∴ Water equivalent of thermometer 25 .	·46	·5	gram.

The last two exercises are examples of the two general methods in use in calorimetry. In the first exercise a quantity of heat was determined by the decrease of temperature of a given mass of water from which it was abstracted, in the second by the increase of temperature of a mass to which it was imparted.

SECTION XIX

Specific Heat. III.—Determination of Specific Heats by the Method of Mixtures

Apparatus required.—Calorimeter, heater and two thermometers.

The exercise in this section consists in finding the specific heat C of a solid, by heating a known mass M to a temperature T, and quickly immersing it in a mass m of a liquid at a temperature t, and which has no chemical action on the solid. If c is the specific heat of the liquid, w the water equivalent of the calorimeter and the thermometer, and θ the final temperature of the mixture, then, equating the quantity of heat lost by the hot body to that gained by the calorimeter, we have

$$C \cdot M(T - \theta) = (c \cdot m + w)(\theta - t).$$

This equation shows us that if the specific heat of either the solid or the liquid is known, that of the other can be found by the above method.

In the following exercise the specific heat of marble is found by mixing it with water the specific heat of which is 1. Putting $c = 1$ in the above equation we find

$$C \cdot M(T - \theta) = (m + w)(\theta - t).$$

An apparatus in which the marble can be heated to a temperature near the boiling point of water is provided.

The marble is placed in a tube A (Fig. 31), which by means of a stopper P is fitted into an outer vessel K, containing water. The stopper should be taken out to see that the outer tube is about one-third full of water, then replaced.

The apparatus is provided with a tube B through which, when the water is heated, the steam escapes. This tube

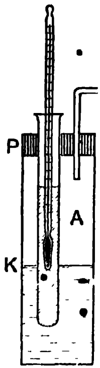


FIG. 31.

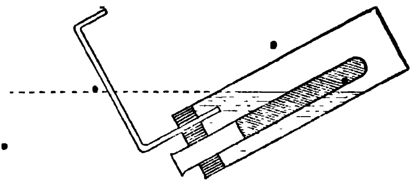


FIG. 32.

should be of such form and length, that when the vessel is tilted, as in Fig. 32, so as to deliver its contents into the calorimeter, no water escapes. Test that this is so before commencing the experiment, then proceed as follows :—

1. Weigh out sufficient marble to fill the inner tube of the heater about two-thirds full, place about one half of the marble in the tube, then put in the thermometer and pack the rest of the marble round it.

2. Place the heater on a retort stand and light the Bunsen burner. Place a can under the end of the bent tube to catch any water coming from it. Lower the flame so as to keep the water just boiling, while the rest of the preparations are being made.

3. Weigh the calorimeter and stirrer. Pour into the calorimeter sufficient water to fill it a little over half full and weigh again. Place in it a thermometer, and observe the temperature when it has become steady.

4. When the water in the heater has been boiling for a few minutes, the temperature of the thermometer in the marble will be found to keep steady somewhere about 99° or 100° C. (possibly if the barometer is very high, slightly above 100° C.). When this has been the case for five minutes, record the temperature. Read and record the temperature of the calorimeter, then take the thermometer out of the heater, remove the heater from the stand with the help of the baize wrapped round it, and quickly empty the marble into the calorimeter, removing the thermometer from the calorimeter for an instant while the marble is being poured in. The heater should be kept as short a time as possible in the neighbourhood of the calorimeter, as it will heat that vessel by radiation.

5. Stir the marble and water in the calorimeter well, and read the temperature to which the thermometer placed in it rises.

6. Repeat the experiment.

Enter and reduce the observations as in the following example of a determination of the specific heat of iron.

Calorimeter used, No. 6.
Thermometer in Heater No. 35.
,, ,, Calorimeter No. 43.

Mass of iron borings (M)	50	55·1	grams
Mass of water in calorimeter (m).	92·2	104·1	,,
Water equivalent of calorimeter and thermometer (previously determined)	5·0	5·0	,,
Total water heated	97·2	109·1	,,
Temperature of iron (T)	99°·1	99·2	C
,, ,, calorimeter (t) . .	18°·2	19·1	
,, ,, mixture (θ). . .	22°·8	23·5	
Specific Heat of iron calculated . .	·117	·118	

Note.—Instead of weighing out the marble and water used separately, we might first weigh the calorimeter empty, then when filled with the water, and finally at the end of the experiment when it contains both water and marble.

As regards the precautions to be taken and the accuracy to be expected, we remark that the rise of temperature in the above example was 4°·6 C. An error of reading of one-tenth of a degree would therefore produce an error of 2 per cent. in the calculated specific heat. This, then, is the accuracy at which we might reasonably aim. The difference of temperature recorded by the thermometer will not be in error to that extent owing to the errors of the thermometer at 0° and at 100°, which therefore need not be known. Similarly the thermometer in the heater will not, as a rule, show an error so great as 1° near the boiling point. If in the above example, the temperature of the iron had been 100° instead of 99°·1 as indicated by the thermometer, an error of about 1·5 per cent. would have resulted, as the iron would have cooled through 77°·2 instead of 76°·3, as assumed. If the error of the thermometer at the boiling point is known, it may, however, be taken into account. The weights should be correct to less than ·1 per cent., that is to say, to about half a gram.

No account has been taken of the loss of heat from the calorimeter by radiation, which takes place after the marble has been introduced and before the final temperature is measured. On the other hand, there is a gain of heat by radiation from the heater, which will tend to counterbalance this loss. See p. 106 for a simple method of applying a correction to the observed results.

The same method of procedure may be adopted in other cases, *e.g.* to find the specific heat *c.* of a liquid, given that *c.* for a solid on which it has no chemical action.

If the two act chemically on each other, the solid may be enclosed in a closed glass or other vessel on which the liquid has no action. The heat absorbed or liberated by the vessel must then be taken into account.

SECTION XX

Latent Heats

Apparatus required.—Calorimeter, thermometer, ice, flask, delivery tube, and condenser.

When a substance is to be converted from the solid into the liquid, or from the liquid into the gaseous state, a certain amount of heat has to be supplied to the substance, which produces no change of temperature. This amount of heat is called the Latent Heat of fusion or of evaporation respectively.

In either case it is generally found by passing the substance in its hotter state into a cool calorimeter, the temperature of which is raised in consequence. From the increase of temperature of the calorimeter the amount of heat imparted to it may be calculated. This amount of heat has been given up by the hot substance in changing its state, and in further cooling to the final temperature of the calorimeter. If the specific heat of the substance is known, the second portion of this heat can be calculated, and the Latent Heat, which is the first portion, may be found by subtraction.

Exercise I

Determination of the Latent Heat of Fusion of Ice or Latent Heat of Water

In this case the usual process is reversed, and the ice at 0° C. introduced into a calorimeter containing water at

a few degrees above the temperature of the room. In consequence of the heat absorbed by the ice in melting, the temperature of the water in the calorimeter falls.

Weigh the calorimeter provided, and place in it sufficient water at about 20° C. to fill it a little over half full. Weigh again to find the water contained. Place a thermometer in the water and record its reading. Select a piece of ice about 10 grams in weight from the ice provided, dry it with blotting paper, and place it, without touching it with the fingers, in the calorimeter. Move it about in the water by means of the stirrer, keeping it all the time under water.

Take readings of the thermometer every half minute till the temperature ceases falling and begins to rise.

Weigh the calorimeter and contents again.

Repeat the experiment.

If M is the mass of ice at 0° C.,
L ,, latent heat of water,
m ,, mass of water in the calorimeter,
t ,, initial temperature of calorimeter,
θ ,, final ,, ,, ,,
w ,, water equivalent of calorimeter and thermometer,

the heat lost by the calorimeter

$$= (m + w)(t - \theta)$$

and the heat absorbed by the ice

$$= \text{M}.\text{L} + \text{M}.\theta = \text{M}(\text{L} + \theta).$$

These two amounts of heats must be equal.

Hence

$$\text{L} = \frac{m + w}{\text{M}}(t - \theta) - \theta.$$

Record as follows:—

Weight of calorimeter	55·1	55·1	grams.
,, ,, ,, and water	201·1	217·2	,,
,, ,, water (m)	146·0	162·1	,,
,, ,, calorimeter and contents at end	214·3	231·2	,,
,, ,, ice added (M)	13·2	14·0	,,

Water equivalent of calorimeter (w) .	5·5	5·5	grams.
Total water ($m + w$)	151·5	167·6	
Initial temperature (t)	22·3	23·3	C.
Final temperature (θ)	14·2	15·4	C.
Latent heat of fusion of ice or latent heat of water	79·0	= 79·6	

EXERCISE II

Determination of the Latent Heat of Evaporation of Water or Latent Heat of Steam

Weigh the calorimeter and stirrer and the condensing vessel provided, then fill the calorimeter to within 2 cms. of the top with water at the temperature of the room, and weigh again.

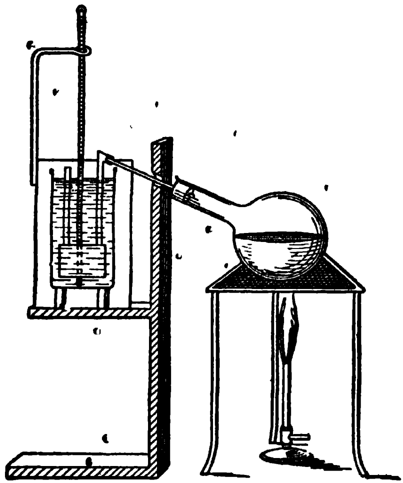


FIG. 33.

Test whether the flask and rubber-tipped delivery tube can be arranged as shown in Fig. 33. The rubber should fit loosely into the end of the condenser, so that the joint can easily be made or broken. It need not be steam tight. Disconnect the condenser from the delivery tube and boil the water in the flask gently, the steam being allowed to escape into the air.

Observe the temperature of the water in the calorimeter, and at a given instant to be noted, place the calorimeter in position, inserting the end of the delivery tube into the top of the tube of the condenser. The steam condenses and the water in the calorimeter rises in temperature. Observe the thermometer every half-minute, keeping the water well stirred, and when the temperature reaches 30° C. remove the delivery tube from the condenser.

Observe the temperature of the calorimeter every half minute for 2 minutes after the highest temperature has been reached, then remove the condenser, dry its outside surface, and weigh it with its contents again.

Write down the equation to determine the Latent Heat of steam, using the symbols L, *m*, *t*, &c., with the meanings given below.

Tabulate the observations as follows :—

First Experiment.		Second Experiment.	
$3^h\ 30^m{\cdot}5$	16°·6 steam on	$4^h\ 25^m{\cdot}5$	13°·0 steam on
$3^h\ 33^m{\cdot}5$	29°·5 steam off	$4^h\ 27^m{\cdot}5$	31°·7 steam off
34^m	31°	28^m	33°·1
$34^m{\cdot}5$	30°·7	$28^m{\cdot}5$	33°·3
35	30°·5	29	33°·0
35·5	30°·4	29·5	32°·8
36	30°·3	30	32°·5
		30·5	32°·3

	First Experiment. grams.	Second Experiment. grams.
Weight of calorimeter empty . .	54·6	55·0
,, ,, ,, and water .	161·9	153·1
Hence mass of water (m) . . .	107·3	98·1
Weight of condenser empty . .	40·0	40·1
Weight of condenser and water after condensation of steam	42·9	43·7
Hence mass of steam condensed (M) .	2·9	3·6
Water equivalent of calorimeter and condenser (w)	9·4	9·5
Total water equivalents	116·7	107·6
Initial temperature of calorimeter (t) .	16°·6	13°·0
Highest temperature of calorimeter (θ)	31°·0	33°·3
Hence latent heat L of steam at 100° C. (uncorrected for cooling) . .	510	531

Students will notice that the temperature continues to rise for a short time after the steam is cut off. This is due to the fact that it takes some time for the water in the condenser to give up its heat till its temperature becomes equal to that of the water in the calorimeter.

It is necessary to point out some sources of error to which these experiments are liable, and for which no allowance has been made. In the first place it was assumed in the calculation that the steam condensed at 100° C., and unless the barometer stands exactly at 76 cms. this will not be correct. Since however a change of three centimetres in the height of the barometer will only change the calculated value of the latent heat by one unit, and other probable errors cause much larger differences, it is not necessary in an experiment of this kind, which does not lay claim to great accuracy, to take account of the variations of the boiling point of water due to changes of pressure.

A considerable error is introduced by the assumption that the heat which enters the calorimeter, all serves to heat up the water, and that none escapes by radiation, conduction, and convection.

In accurate calorimetric experiments a so-called "cooling correction" has to be applied to the final readings of the thermometer, to give the temperature which *would have been observed* if all the heat had been retained in the calorimeter. The methods of finding and applying this correction are outside the range of this book, but we shall in the present instance show how a rough estimate of it may be made. It is for this purpose that students have been instructed to read the thermometer after it has reached its maximum. Referring back to the examples given, it will be found that in the first experiment the thermometer fell 0°·7 during the next two minutes after reaching its highest point. This fall indicates that the amount of heat lost by the

calorimeter owing to convection, &c., was sufficient to lower its temperature 0°·7 in two minutes. The interval of time between the introduction of steam, and the thermometer reaching its highest reading, was $3\frac{1}{2}$ minutes, and if during the whole of that time, the loss of heat had been the same as at the end of the experiment, the heat lost in the $3\frac{1}{2}$ minutes would have caused a fall of the thermometer amounting to $\frac{0°\cdot7 \times 3\cdot5}{2} = 1°\cdot2$. But the heat lost by the calorimeter depends on the excess of its temperature above that of the surroundings. At the beginning of the experiment the calorimeter was at the same temperature as the air surrounding it, and therefore no heat was lost, but as the temperature of the calorimeter rose higher, more and more heat was given up. If the amount of heat lost by radiation increases in proportion to the rise of temperature, the average loss per unit time over the whole interval, is only half the loss per unit time at the end of the experiment. Approximately, therefore, the loss of heat due to radiation, from the time steam was introduced, to the time the thermometer reached its highest point, is only half the amount stated above, *i.e.* it produced a fall of temperature of 0°·6, which is the cooling correction required. In this experiment therefore the highest temperature reached would have been 31°·6 if the calorimeter had been protected against all loss. Similarly in the second experiment the highest reading of the thermometer would have been 33°·7.

Correct the observations in this manner for cooling, and calculate the latent heat with the corrected temperatures, entering the results as follows:

Highest temperature of calorimeter corrected for cooling	31°·6	33°·7
Latent heat of steam (corrected) . . .	536	554

This exercise on the latent heat of steam has been introduced here, as it forms a useful exercise in calorimetric

work, but accurate results cannot be expected unless very great precautions are taken. The simple apparatus here described is found to give more consistent results than other forms, in which the steam is led directly into the water without a condenser being used. The results may be rendered more accurate by weighing the condenser empty and with the water condensed in it apart from the calorimeter in a more delicate balance.

The first experimemt given above, has led to a result which is very near the truth, while the value obtained in the second experiment, differs rather more than usual from the correct number. Students should have no difficulty in obtaining results which do not show errors greater than 3 per cent.

As the condensed vapour is not brought into contact with the liquid of the calorimeter, the same apparatus and method may be used to determine the latent heat of vaporisation of any liquid.

If the latent heat of vaporisation L, the specific heat C, and the boiling point T of a liquid are given, the same apparatus and method would serve to determine the specific heat c of any liquid used as condensing liquid in the calorimeter. The equation connecting these quantities with the initial temperature t and final θ of the calorimeter being:

$$M(L + C.\overline{T - \theta}) = (mc + w)(\theta - t).$$

SECTION XXI

Melting and Boiling Points

Apparatus required.—Test-tube with naphthalene, two thermometers, flask, carbon tetrachloride, water bath, and condenser.

If a crystalline solid is heated, it is converted at a certain well-marked temperature into a liquid, which if allowed to cool, solidifies again at the same temperature. This temperature is called the melting point of the solid, or the freezing point of the liquid. If the solid is non-crystalline or amorphous, like wax and the fats, the change from solid to liquid, or *vice versâ*, takes place gradually, so that it is difficult to state at what temperature the solid melts, or the liquid freezes.

Exercise I

Determination of the Melting Point of Naphthalene

The test-tube provided contains a quantity of naphthalene, in which the bulb of the thermometer is embedded.

Support the test-tube in a clamp, and place under it on a tripod, a beaker containing water. Lower the test-tube into the beaker, till the water outside reaches up to the level of the surface of the naphthalene inside the tube. Place a thermometer in the water in the beaker.

Raise the temperature of the water to 70° C., then lower the flame, and continue the heating slowly, watching the naphthalene carefully, to see when it begins to melt

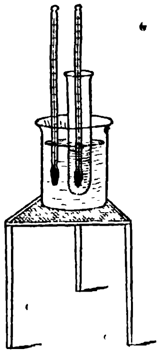


FIG. 33a.

at points in contact with the walls of the test-tube. When this occurs note the temperature of the water and keep it constant by further lowering the flame or removing it entirely. Keep the thermometer and mass of unmelted naphthalene attached to its bulb in motion, and observe the reading on it every half-minute, noting that it remains nearly constant while the solid is melting.

When the naphthalene round the bulb of the thermometer begins to melt, and allows the mercury to be seen, observe the temperature, and take this temperature as the melting point. Observe the temperature for three minutes, during which it will be found to rise.

Now remove the beaker, dry the outside of the test-tube, and allow it to cool by losing heat to the air, observing the temperature every half-minute till the substance is solid. The temperature will be found to remain constant while the liquid is solidifying, but will decrease at other times. Take the constant temperature as the freezing point of the liquid.

Compare the two results obtained as follows :—

Melting point	=78°·8 C.
Freezing point	=78°·7 C.

Draw a curve showing the rise and fall of temperature, from six half-minute intervals before, to six half-minutes after, the melting and freezing respectively.

In the same way if heat is supplied to a liquid, at a definite temperature depending on the atmospheric pressure at the time of observation, the liquid is converted into vapour, remaining at the same temperature till the whole of it is vaporised. The temperature indicated by a thermometer in the liquid itself, is slightly influenced by dissolved salts and by the nature of the vessel, but that indicated by a thermometer in the steam, is dependent on the liquid and the pressure only.

This latter temperature is called the boiling point of the liquid under the observed pressure.

Exercise II

Determination of the Boiling Point of Carbon Tetrachloride.

Pour sufficient carbon tetrachloride into the flask provided to form a layer about 2 cms. in thickness, and support the flask on a shallow water bath in such a way that the level of the surface of the water outside, is a little above that of the liquid inside. Insert into the flask through the cork a thermometer, the bulb of which should be about 2 cms. above the surface of the liquid, and a delivery tube, the other end of which passes into the neck of a small flask, placed in cold water to serve as a condenser. (Fig. 34.)

Heat the bath slowly, observing the temperature every minute. When the liquid begins to boil, turn down the flame so that the boiling goes on gently. Notice that the indication of the thermometer remains constant. The

temperature indicated is the boiling point of the liquid under the atmospheric pressure at the time, which should be read on the barometer. Reduce the observation to

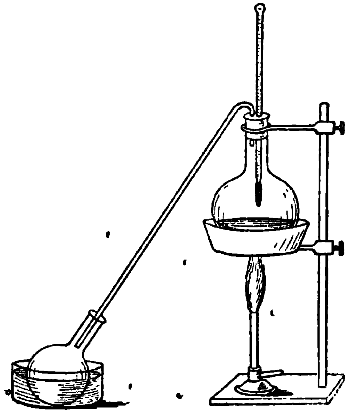


FIG. 34.

76 cms. pressure by using the known fact, that the boiling point of carbon tetrachloride changes 0°·4 C. for a variation of 1 cm. in the pressure.

Observed boiling point at	74·9 cms.	76°·7 C.
Calculated ,,	76·0 ,, ...	77°·1 C.

SECTIONS XXIA AND XXIB

These sections appear as Appendix A, page 230.

PART IV.

LIGHT

SECTION XXII

Reflection at a Plane

Apparatus required.—Drawing board, mirror and support, sighting rod, and cm. scale.

Formation of Images by a Plane Mirror

Let Q be a small object in front of a reflecting surface

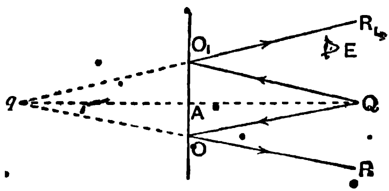


FIG. 35.

AO (Fig. 35). Draw rays QO, Q_1O, &c. from Q. These will be reflected according to the laws of reflection, along OR, O_1R_1, so that in each case the incident and reflected rays and the normal to the reflecting surface at the point of incidence are in the same plane, and the two rays make equal angles with the normal on opposite sides of it. It can be proved by elementary geometry that these reflected rays when produced backwards, will all intersect in a point *q*, such that a line *q*Q is at right angles to the reflecting surface, and the point *q* as far behind the surface as Q is in front.

To an eye placed at any point E, the reflected rays will therefore appear to come from q, and the point q is called the image of Q.

EXERCISE.

To verify that the image is as far behind a plane reflecting surface as the object is in front.

(1) *By the sighting method.*

Place a pin Q vertically in front of a strip of mirror A which is supported on a horizontal drawing board by a clip so that its reflecting surface is vertical (Figs. 36 and 38)

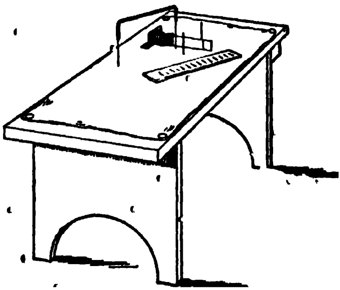


FIG. 36.

To find the position of the image q, use the apparatus shown in Fig. 37. P_1 P_2 is a bent rod with two needles

FIG. 37.

or pins at its ends parallel to each other. Shut one eye and place P_1 P_2 in such a position (Fig. 38), that when the open eye is about 20 cms. behind P_2, the points of the

pins P_1 P_2 and q are in the same straight line. Mark the positions of P_1 and P_2. Repeat the experiment with the eye in different positions. Draw a line on the paper along the *silvered* surface of the mirror at which the reflection

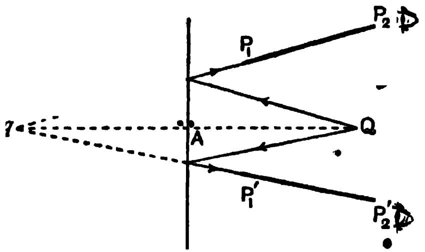


FIG. 38.

takes place, and take away the mirror. The lines joining the points P_1 and P_2, P_1' and P_2', &c., should, when produced, all intersect in the same point q, and qA and QA will be found to be nearly equal to each other. Owing to the refraction of the light through the glass in front of the silvered surface, the image will be about $\frac{2}{3}$ of the thickness of the glass nearer to the reflecting surface of the mirror than the calculated position. (*See* Fig. 42, p. 122.)

(2) *By the parallax method.*

Place the mirror A and pin Q as before. The image of Q will be q (Fig. 39). Look straight at the mirror with one eye only, placed so that the pin Q nearly covers the

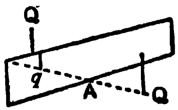


FIG. 39.

image q. Place a pin Q′ vertically, so that the upper part of it appears continuous with the image q of the lower part

of the pin at Q, seen in the mirror. Q′ will then be on the line passing though q and the eye, but it may be in front of or behind q. Now move the eye to the left so that the mirror is looked at obliquely. If the needle Q′ appears to the right of the image q, it is too near the mirror, if to the left as in the figure, it is too far away. In the former case move it a few millimetres further away from the mirror in a direction perpendicular to the mirror. Place the eye again so that Q nearly covers its image q, and see that Q′ still appears to be continuous with q. Move the eye again to the left and see whether the two still appear continuous. If not, move the pin again, and repeat the observations. By this method, a position of the pin will be found such that, whatever the direction in which the eye looks, the image q of the lower part of the pin Q, and the upper part of the pin Q′ appear to be continuous. The shifting of the image q and the pin Q′ over each other, when the latter is not in its proper position, is called parallax. When there is no longer parallax, draw a pencil line on the paper to mark the position of the silvered surface of the mirror. Measure the distances of the pins from this line by means of a glass millimetre scale, laid on the paper with the graduations on the under surface, or with a wooden scale placed on edge so that the graduations extend to the surface of the paper.

Repeat the experiment with Q at different distances from the mirror.

The student should make a reduced copy in his note-book of the lines on the drawing paper, and state the observed lengths of AQ, Aq, &c., in each case.

SECTION XXIII

Refraction at a Plane

Apparatus required.—Drawing board, glass cube, sighting rod, and drawing instruments.

Explanation of the Laws of Refraction

Let IO (Fig. 40) be a ray of light in air, which falls at O on a surface MM′ of glass or water.

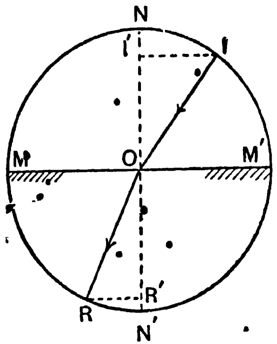


FIG. 40

With centre O describe a circle IMM′, draw the normal NON′ to the surface MM′, and II′ perpendicular to ON. The ray IO on entering the lower medium, which is optically denser than the upper, will continue in the plane containing ON and OI (first law of refraction), but will be bent towards the normal NN′.

Let OR be the refracted ray, then the angle IOI′ is called the angle of incidence, and the angle ROR′ the angle of refraction. Through R the point of intersection

of the refracted ray and the circle draw the line RR′ perpendicular to the normal and intersecting the normal in R′.

It has been found experimentally, that the ratio of the lines II′ and RR′ to each other is the same whatever the angle of incidence. If the first medium is air, this ratio is called the refractive index μ, or index of refraction of the second medium. The index of refraction is different for rays of different colours, increasing from red through the yellow, green, and blue to the violet. The following table gives for several media the approximate refractive indices for yellow light :—

Diamond	2·44 to 2·75
Flint glass	1·58 to 1·64
Crown glass	1·53 to 1·56
Bisulphide of carbon	1·68
Water	1·33

We may therefore take the index of refraction of crown glass as $\frac{3}{2}$, and of water as $\frac{4}{3}$.

The ratio II′ to OI in the right-angled triangle IOI′ of the above figure, is called the sine of the angle IOI′. Similarly the ratio RR′ to OR is the sine of the angle ROR′.

Hence :—

$$\frac{\text{Sine angle of incidence}}{\text{Sine angle of refraction}} = \frac{\frac{II'}{OI}}{\frac{RR'}{OR}} = \frac{II'}{RR'} \times \frac{OR}{OI}$$

$$\text{but } OR = OI, \text{ and } \frac{II'}{RR'} = \text{refractive index.}$$

$$\therefore \frac{\text{Sine angle of incidence}}{\text{Sine angle of refraction}} = \text{refractive index.}$$

This is the second law of refraction, called after its discoverer Snell's Law.

Exercise I

To verify Snell's Law

Place the cube provided on the paper with the line drawn along one surface parallel to an edge vertical. By

means of the sighting rod P_1 P_2 (Fig. 41), sight the lower part of the line R from three positions on the opposite

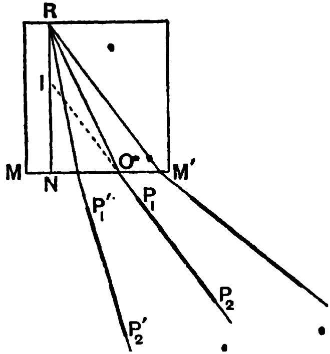


FIG. 41.

side of the cube to the line. The directions of the three emergent rays P_1P_2, $P_1'P_2'$, &c., are thus obtained. Mark on the board, MM′ the projection of the refracting surface, and R the projection of the vertical line on the cube, and take away the cube. Produce P_1P_2 backwards until it meets the face of the cube in O. Join RO. RO will be the ray incident on MM′, which on emergence passes through P_1P_2. Draw RN normal to MM′, and produce P_2O to intersect RN in I.

By a general law in Optics, the course of a ray can be reversed without altering its path, hence if the ray going from R to P_2 takes the path RO, OP_2 a ray going from P_2 to R would take the path P_2O, OR. In the latter case, since RN is normal to the surface MM′, OIN would be the angle of incidence, and ORN the angle of refraction. Hence by the law of refraction

$$\frac{\text{sine OIN}}{\text{sine ORN}} = \text{refractive index},$$

$$\text{but sine OIN} = \frac{\text{ON}}{\text{OI}}, \text{ and sine ORN} = \frac{\text{ON}}{\text{OR}}.$$

$$\text{Hence index of refraction} = \frac{\text{ON}}{\text{OI}} \times \frac{\text{OR}}{\text{ON}} = \frac{\text{OR}}{\text{OI}}.$$

Measure OR and OI, calculate their ratio, repeat for rays at other inclinations to the surface MM′, and record as follows :—

Cube, No. 20.

OR.	OI.	OR/OI = μ.
4·49	3·00	1·50
4·79	3·18	1·51
4·98	3·30	1·51

If it is found that OR/OI is the same whatever the direction of the emergent ray, Snell's law will have been verified.

It will be found that the various sighting lines P_1P_2, $P_1'P_2'$, &c., when produced backwards, intersect nearly at the same point I on the line RN. This is due to the fact that if the point O is not far from N, the ratio OR to OI is nearly equal to the ratio NR to NI, which ratio is therefore nearly equal to the refractive index.

The student should draw a reduced diagram in his notebook, and should give the lengths of all the lines measured, and the refractive index calculated from each pair, as above.

We can now understand why in reflection at an ordinary

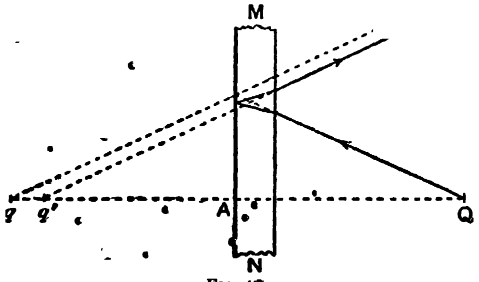


FIG. 42.

mirror, the image is not quite so far behind the reflecting surface, as the object is in front. The rays from Q (Fig. 42) are refracted at the surface of the glass on entering and emerging, and this causes the reflected ray to appear to come from q', and not from q, where $Aq = AQ$.

EXERCISE II

Geometrical Construction for the Refracted Ray

It has been seen that if the index of refraction of the denser medium is μ, the line II′ (Fig. 40) is μ times RR′, hence if the ray IO and the index μ were given, the direction of the ray OR could be found by measuring along OM a length equal to II′ divided by μ, and letting fall from the end of this length, a perpendicular to MM′, cutting the circle IMM′ in R. This method involves drawing for each incident ray the perpendicular II′, measuring it, and cutting off from OM a length equal to II'/μ. The following method is more convenient.

With centre O (Fig. 43) draw two circles with radii proportional respectively to the indices of refraction of the two media, *i.e.* 1 and μ. Let the ray IO, traversing the medium of index 1 cut the circle of radius 1 in I. Let fall from I a

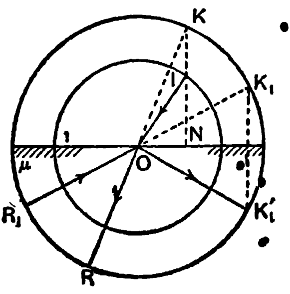


FIG. 43.

perpendicular IN to the surface of separation of the two media, and produce this perpendicular upwards to cut the

circle of radius μ in K. Join KO and produce to R; OR is the refracted ray in the medium of index μ.

For, sine OIN = ON/OI, and sine OKN = ON/OK

$$\therefore \frac{\text{sine OIN}}{\text{sine OKN}} = \frac{\text{ON}}{\text{OI}} \cdot \frac{\text{OK}}{\text{ON}} = \frac{\text{OK}}{\text{OI}} = \mu.$$

and angle OIN = angle of incidence
,, ,, OKN = ,, ,, refraction.

Hence the construction makes the sines of these angles have the proper ratio to each other.

The student should draw in his note-book, using the same pair of circles, the refracted rays corresponding to rays incident at angles of 10°, 20°, &c., 80°, on the surface of a medium of which the index of refraction is 1·55, and should notice that the refracted rays due to rays incident at angles nearly 90°, make with the normal angles much less than 90°.

If we consider rays in the dense medium, to be incident at O from all directions, only those which make an angle with the normal less than a certain limit, will be refracted into the rare medium. This limiting angle is called the Critical Angle, and the rays which are incident at angles greater than the critical angle, are entirely reflected, remaining in the dense medium and constituting the "totally reflected rays." So long as an incident ray is capable of producing a refracted ray, the construction given above, carried out in the reverse order, will give the direction of that refracted ray, but it will be found to fail when the angle of incidence in the denser medium is greater than a certain limit, as in the case of the ray R_1O (Fig. 43). The normal through K_1, the point in which the ray produced cuts the circle of radius μ, does not cut the circle of radius I, and must be produced downwards to cut the outer circle again in K_1', and K_1' be joined to O. OK_1' is then the totally reflected ray corresponding to the incident ray R_1O.

The student should draw the refracted rays produced by rays incident at 10°, 20°, &c., 80° on the inner surface of a medium of index = 1·55.

SECTION XXIV

Lenses and Mirrors. I

Apparatus required.—Drawing instruments.

Lenses are transparent bodies, bounded by two surfaces which are generally spherical.

The line joining the centres of the two spheres of which those surfaces are parts, is called the axis of the lens.

We distinguish two kinds of lenses, according to their action on a parallel beam of light.

I. Lenses which change a parallel beam of light into a convergent one, as in Fig. 44, are called converging lenses.

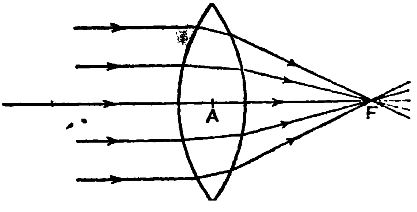


FIG. 44.

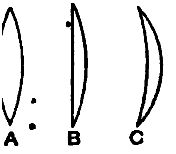


FIG. 45.

Converging lenses always have their thickest part in the middle. The three lenses given in Fig. 45 are converging

lenses. One of the bounding surfaces is always convex to the outside, the other may be convex (A), plane (B), or concave (C); but if it is concave, the curvature of the convex surface must be greater than that of the concave one.

The point to which the rays of light falling on the lens parallel to the axis, converge after refraction, is called a principal focus. There are two principal foci, one on each side of the lens, since a parallel beam of light may fall on the lens from the left or from the right. If the media on the two sides of the lens are the same and the lens is thin, the two foci will be at equal distances from the lens.

The distance of either focus from the lens is called the focal length of the lens. The geometrical constructions and formulæ which are given in this Section, apply only when the thickness of the lens is so small compared to the focal length, that it becomes immaterial whether the focal length is measured from the surface of the lens, or from some point inside. If the figures which follow were accurately drawn to scale, the lens would appear almost as a line. In order to distinguish convex and concave lenses, we shall draw the lenses with an exaggerated thickness, but in dotted lines; while the position of the actual lens is given by a straight line drawn in full; thus in Fig. 46 the lens is really

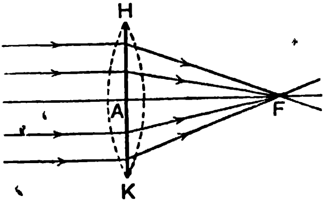


Fig. 46.

represented by the line HK, and the two curved lines only indicate that the lens is a convex one. The incident and

refracted rays are therefore drawn straight up to the line HK. The same holds in all the figures which follow.

II. Lenses which change a parallel beam of light into a divergent beam, as in Fig. 47, are called diverging lenses.

Diverging lenses always have their thinnest part in the middle, as in Fig. 48. One of the bounding surfaces is

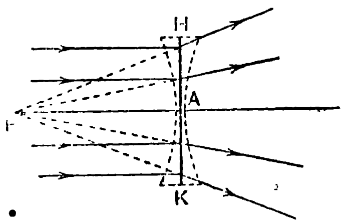


FIG. 47.

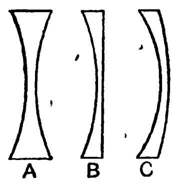


FIG. 48.

always concave, the other may be concave (A), plane (B), or even convex (C); but in the last case the concave surface must have a greater curvature than the convex one.

The point F (Fig. 47), from which the beam of light seems to diverge after refraction, is called a principal focus, the other focus being at the same distance from the lens on the opposite side.

Converging lenses are sometimes called convex, and diverging lenses sometimes concave, lenses.

If a pencil of light diverging from a luminous point Q, fills a lens HK (Fig. 49), the quantity $1/_{QA}$ is called the divergence of the incident pencil with respect to the lens. If the pencil after refraction converges to Q′, the quantity $1/_{Q'A}$ is called the convergence of the emergent pencil.

Since the medium on the two sides of the lens is the same, there are two foci, F_1 and F_2, on opposite sides

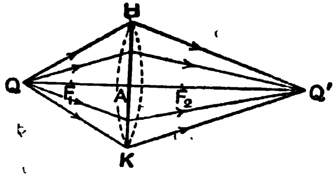


FIG. 49.

of the lens and at equal distances from it, and the quantity $^1/_{AF_1}$ or $^1/_{AF_2}$ is called the converging power of the lens. In order to make our statements definite, we take for the point A, the point at which the plane passing through the edge of the lens HK cuts the axis; but the relations which are given below between the distances of the object and image, are only approximate, and the errors committed are of the same order of magnitude as the thickness of the lens, so that these distances might, without appreciable difference in the error, be measured from the surface of the lens. The distance of the image from the lens can be calculated from the following proposition, which is the more nearly correct the thinner the lens.

Convex lenses increase the convergence, or diminish the divergence, of an incident beam by a constant quantity, which is equal to the converging power of the lens.

In order to show how this proposition is applied, we distinguish three cases.

(1) If the luminous point is nearer to the lens than the principal focus (Fig. 50), the pencil after passage through the lens, will still be divergent, but its divergence will be diminished.

The divergence of the incident beam is $^1/_{AQ}$, the converging power of the lens is $^1/_{AF_1}$. Hence, the divergence of the

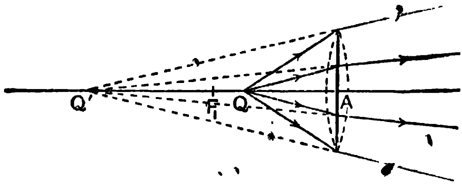


FIG. 50.

emergent beam is $^1/_{AQ} - {}^1/_{AF_1}$. Thus, according to the above proposition,

$$\frac{1}{AQ'}=\frac{1}{AQ}-\frac{1}{AF_1}.$$

(2) If the luminous point is further from the lens than the principal focus (Fig. 49), the converging power of the lens will be greater than the divergence of the pencil, and the emergent pencil will therefore be convergent, the convergence being given by the equation :—

$$\frac{1}{AQ'}=\frac{1}{AF_1}-\frac{1}{AQ}.$$

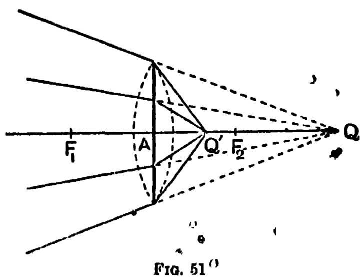


FIG. 51.

(3) If the incident beam is convergent, its convergence will be increased by the lens (Fig. 51). Hence:—

$$\frac{1}{AQ'} = \frac{1}{AQ} + \frac{1}{AF_1}.$$

Geometrical Constructions

Let F_1AF_2 (Fig. 52) be the axis of the lens, F_1 and F_2 the focal points, A the centre of the lens, and let Q be a point on

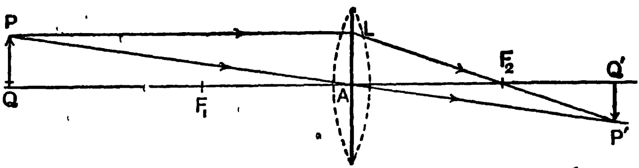


FIG. 52

the axis to the left of the lens and PQ a line perpendicular to the axis. It is required to find the position and size of the image of PQ.

Through P, draw two rays, one PA through A the centre of the lens, and produced in the same direction after passing through the lens, the other PL parallel to the axis, meeting the plane through A perpendicular to the axis, in the point L. This ray, after refraction at the lens, passes through F_2. The image of P will be at the point of intersection P′ of the two rays LF_2 and PA. This point of intersection may be either on the F_2 or right side of the lens, in which case the image is real and inverted, or on the F_1 or left side (Fig. 53), in which case the image is virtual and erect. The first will be found to be the case if Q is to the left, the second if it is to the right of F_1. Through

P′ draw P′Q′ perpendicular to the axis. P′Q′ is the image of PQ. If the rays LF_2 and PA have to be produced to the left to intersect, draw the portions LP′, PP′ of them to the left of the lens, and the image P′Q′, dotted. By considering each ray in this case to be reversed in direction,

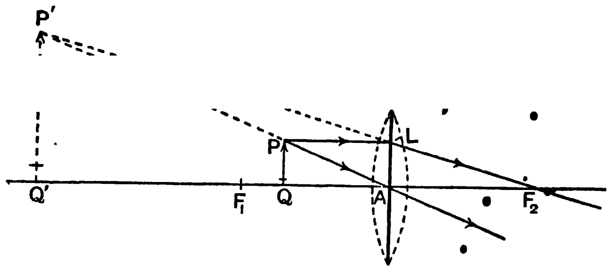


FIG. 53.

the construction gives the position of the real image PQ produced by rays which if the lens were not present, would form an image P′Q′.

Work out the following examples by the convergence and divergence method, and by the geometrical method, giving diagrams :—

A convex lens has a focal length of 12 centimetres. Find the positions of the images corresponding to objects at distances of 60, 24, 8, 6, 2 centimetres respectively to the left of the lens.

The position of the image produced by a concave lens, if the position of the object is given, can be found with the help of a proposition similar to one given above.

Concave lenses increase the divergence, or diminish the convergence, of an incident beam by a constant quantity, which is equal to the diverging power of the lens.

Write out the equations which determine the positions of the images in the three following cases:

(1) The incident pencil is divergent.

(2) The incident pencil converges to a point beyond the lens, but nearer to the lens than the principal focus.

(3) The incident pencil converges to a point beyond the lens and further from the lens than the principal focus.

Find the positions of the images formed by a concave lens of focal length 12 cms. of objects placed at 60, 24, 8, 6 and 2 cms. to the left of the lens.

Carry out also the geometrical construction for each case, remembering that parallel rays incident on the *left* surface of a concave lens, appear on emergence to proceed from the focus on the *left* of the lens.

Spherical mirrors are of two kinds:—

I. Those which convert a parallel beam incident on them into a convergent beam. They are concave towards the incident beam, and their converging power, *i.e.*, the reciprocal of the focal length, which is the distance from the mirror to the point to which the parallel beam converges after reflection, is equal to twice the reciprocal of the radius of curvature of the mirror.

Converging mirrors increase the convergence or diminish the divergence of a beam they reflect by an amount equal to the power of the mirror.

II. Those which convert a parallel beam incident on them into a divergent beam. They are convex towards the incident beam, and their diverging power is equal to twice the reciprocal of their radius of curvature.

Diverging mirrors increase the divergence or diminish the convergence of a beam they reflect by an amount equal to the power of the mirror.

Work out the examples set for lenses, replacing everywhere convex or concave lens by concave or convex mirror respectively.

SECTION XXV

Lenses and Mirrors. II

Determination of the Focal Lengths of Lenses and Mirrors.

Apparatus required.—Drawing board, a convex and a concave lens, a convex and a concave mirror, slit, stop, screen, and sighting rod (Fig. 54).

EXERCISE I

To find approximately the focal length of a convex lens, arrange the lens and a small screen with their centres above a straight line ruled on a sheet of paper fixed to

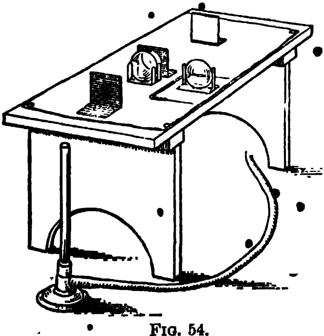


FIG. 54.

the drawing board. Incline the board so that light from a window or from a distant object outside, may pass through

the lens and fall on the screen. Vary the distance of the screen from the lens until a distinct image of the object is obtained. Measure accurately the distance from the centre of the lens to the screen. This will be the focal length, if the lens is thin. Record as follows :—

Lens No. 10.
Focal length from image of object outside = 7·4 cms.

Exercise II

At the end of the board, near one edge of the paper, place a pin upright, and about 10 cms. away from the board place a luminous burner so that the pin may be well illuminated. Draw a line across the paper passing through the pin-hole. Place the centre of the lens vertically above this line about 10 cms. behind the pin, and place the screen at the other end of the line. Adjust the positions of lens and screen until a distinct image of the pin is obtained on the screen.

Measure the distance of the pin from the lens (u), and of the screen from the lens (v), and calculate the focal length as in the following table :—

Lens No. 10.

Experiment.*	u cms.	v cms.	Divergence of incident beam. $1/u$	Convergence of refracted beam. $1/v$	Converging power of lens.	Focal length cms.
1	9·05	36·9	·110	·027	·137	7·3
2	12·55	17·7	·079	·056	·135	7·4
3	17·20	12·75	·058	·078	·136	7·4
				Mean	. . .	7·4

* These numbers may be found most easily by reference to the table of reciprocals on p. 240.

The numbers in the last column should be nearly equal to each other.

Notes.—1. Care should be taken to place the plane of the lens at right angles to the line joining object and image, otherwise the image will have coloured edges, and will be nearer the lens than it should be normally. A considerable error may thus be introduced.

2. Even with the lens properly placed the image may be indistinct and surrounded by a coloured fringe, which is due to the fact that the focal length is different for light passing through different parts of the lens (spherical aberration), and for differently coloured rays (chromatic aberration). By inserting coloured glasses between the light and the object, the trouble arising from chromatic aberration can be diminished; but the determination of the focal length is then more difficult as the light is weakened. By inserting stops of different diameters the effects of spherical aberration may be diminished and the image made much sharper.

For the pin substitute the slit provided, placing it vertical in the position occupied by the pin. Place the screen in such a position that the image is most distinct, then insert the stop with the wider opening so as to block out the light coming through the edges of the lens, and notice that the image becomes sharper. Insert the stop with the smaller opening, and observe that the position of the screen for which the image is most distinct can now be found with greater accuracy. Take observations of u and v, and tabulate the results as before.

EXERCISE III

When the object is nearer to a convex lens than the principal focus, no real image is formed; but when the eye is placed on the opposite side of the lens to the object, on looking through the lens a virtual image on the same side of the lens as the object, is seen. In order to find its position, the sighting rod, or the parallax method may be used. Place the slit half way between the principal focus and the lens, place the flame behind the slit, and look at the image from two positions on the opposite side of the lens to the slit, one on each side of the axis, using the sighting rod as in the previous exercises, to find the position of the virtual image. Measure the distances of the object (u) and image (v) from the lens. Repeat the experiment with the slit placed at a distance from the lens equal to about two-thirds of the focal length. Calculate the results as in the following example :—

Lens No. 10.

Experiment.	u cms.	v cms.	Divergence of incident beam. $1/u$	Divergence of refracted beam. $1/v$	Converging power of lens.	Focal length. cms
1	3·90	8·55	·256	·117	·139	7·2
2	5·80	25·6	·172	·039	·133	7·5
				Mean	. .	7·3

Exercise IV

The image formed by a concave lens is virtual, and cannot be projected on a screen; its position can, however, be determined by the use of the sighting rod.

Substitute for the convex lens of Exercise III. a concave lens, and take two observations with the slit about 25 and 35 cms. from the lens. Calculate out and arrange the results as in the following example:—

Lens No. 7.

Experiment.	u cms.	v cms.	Divergence of incident beam. $1/u$	Divergence of refracted beam. $1/v$	Diverging power of lens.	Focal length. cms.
1	36·4	7·9	·027	·127	·100	10·0
2	24·0	7·1	·042	·140	·098	10·2
				Mean	. .	10·1

Exercise V

Verify the result obtained for the concave lens, by placing it in contact with a convex lens of less focal length, and determining the focal length of the combination by method I. The power of the concave lens will be equal to the difference between the powers of the convex lens and of the combination. Record as follows:—

Focal length of convex lens = 7·4 ∴ converging power = 1/7·4 = ·136

" " " combination = 27·0 ∴ " " = 1/27·0 = ·037

∴ Diverging power of concave lens No. 7 . = ·099

Focal length of concave lens No. 7 = 10·1 cms.

Exercise VI

Determine the focal length of a concave mirror by substituting it for the convex lens, and a small screen about 1 cm. diameter for the screen used in Exercise I, proceeding as described in that exercise.

Exercise VII

Determine the focal length of the concave mirror by placing a pin or slit, about 20 cms. away from the mirror and a few cms. on one side of the axis. Place the screen so that the edge of it is in the axis, and move it to or from the mirror till a distinct image is seen on it. Proceed as in Exercise II, then calculate the focal length of the mirror from the distances of object and image from it.

Exercise VIII

Determine the focal length of the concave mirror by the method described in Exercise III, placing the pin or slit nearer to the mirror than the focus, and using the sighting rod.

Exercise IX

Determine the focal length of the convex mirror by the method described under Exercise IV.

SECTION XXVI

Lenses and Mirrors. III

Experimental Verification of the Relation between the Sizes of an Object and its Image formed by a Lens, and by a Concave Mirror.

Apparatus required.—Drawing board, slit, lens, concave mirror, stop, screen, and instruments.

Let A_1 (Fig. 55) be the centre of a converging lens, and let Q' be the image of a point Q. Then since $1/A_1Q$ is the divergence of the incident beam, and $1/A_1Q'$ the conver-

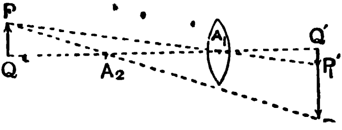


FIG. 55.

gence of the emergent beam, the relation between the positions of image and object is given by the equation

$$\frac{1}{A_1Q}+\frac{1}{A_1Q'}=\frac{1}{f} \qquad (1)$$

where $1/f$ is the converging power of the lens.

Let PQ be a small linear object placed at right angles to the axis of the lens, and let P'_1Q' be the image. When

the lens is thin, the line joining P and P_1' will pass through the point A_1, and the triangles A_1PQ and $A_1P_1'Q'$ will be similar. Hence :—

$$\frac{P_1'Q'}{PQ} = \frac{A_1Q'}{A_1Q} \quad . \quad . \quad . \quad . \quad . \quad (2).$$

or, expressed in words, the *ratio of the sizes of image and object is equal to the ratio of their distances from the lens.*

Exercise

In order to verify equation (2), perform the following experiments :

1. Take a convex lens and determine its focal length approximately, by holding it not less than 5 metres away from a source of light a flame or a window, and finding the position of a screen behind the lens, when the image of the source on it is most distinct.

Measure the distance from the lens to the screen. This is the focal length approximately.

2. Measure the length (a) of the slit in the screen provided, and place the screen near the left-hand edge of the drawing board, with the slit horizontal. At a distance from the screen 2 or 3 cms. more than twice the observed focal length, place the lens in its support. Place the stop provided near the lens, so that the light passes through the centre of the lens only. Put a luminous flame behind the slit, and find the position of the screen when the image of the slit on it is most distinct. Mark the positions of slit, lens and screen.

Measure the distances of the slit (u) and screen (v) from the lens.

3. Measure the length (b) of the image of the slit by means of compasses or a glass scale.

Keeping slit and lens fixed repeat the adjustment of the screen twice, marking the position found and measur-

ing the length of the image of the slit each time. Take the mean of the two sets of observations.

4. Move the screen till its distance from the slit is less than four times the observed focal length of the lens. Notice that there is now no position of the lens in which a good image appears on the screen.

5. Place the screen near the right-hand edge of the board.

There are now two positions of the lens, in each of which a distinct image of the slit is projected on the screen. Measure the distances of the slit and screen from the lens and the size of the image of the slit in each case, making the adjustment twice, and taking the means of the measurements.

Experiment thus shows that for a given distance between an object and a screen there are two positions of the lens such that an image of the object is thrown on the screen, provided the distance between the object and the screen is more than four times the focal length of the lens.

If with the screen at Q′ (Fig. 55), A_1 is a position of the lens for which a sharp image of an object at Q is formed, it may be easily proved that the second position of the lens is at A_2, such that $QA_2 = Q'A_1$. In that case P_2Q_1 will be the image of PQ.

Now

$$\frac{Q'P_2}{QP} = \frac{Q'A_2}{QA_2} = \frac{QA_1}{Q'A_1}.$$

Also

$$\frac{QA_1}{Q'A_1} = \frac{QP}{Q'P_1},$$

hence

$$\frac{Q'P_2}{QP} = \frac{QP}{Q'P_1}$$

$$QP = \sqrt{Q'P_1 \, Q'P_2}.$$

Expressed in words this means that if the object and screen are kept at the same distance from each other, and the two positions of the lens are found such that images of the object are projected on to the screen, the geometrical mean of the lengths of the images is equal to the length of the object.

Record as follows:—

Focal length of lens = 8·1 cms.
Length of slit (a) = 2·0 „

u	v	$u+v$	v/u	b	b/a
18·1	14·7	32·8	·80	1·60	·80
33·3	10·7	44·0	·32	·65	·33
10·6	33·4	44·0	3·15	6·25	3·13

Geometrical mean of the two values of b, for a distance of 44 cms. between slit and screen $= \sqrt{6{\cdot}25 \times {\cdot}65} = 2{\cdot}01$.

Proceed in the same way to verify that for the concave mirror the sizes of object and image are proportional to their distance from the mirror. The slit and screen will now both be on the concave side of the mirror, and slightly displaced in opposite directions out of the axis of the mirror.

SECTION XXVII

Refraction of Light through a Prism

Let AI (Fig. 56) be a ray of light incident on a prism, the refracting angle of which is a. It will be refracted along IR, and if i and r are the angles between the normal NI and the rays outside and inside the prism respectively, we have by the laws of refraction

$$\sin i = \mu \sin r \quad . \quad . \quad . \quad . \quad (1)$$

where μ is the index of refraction of the material of the prism.

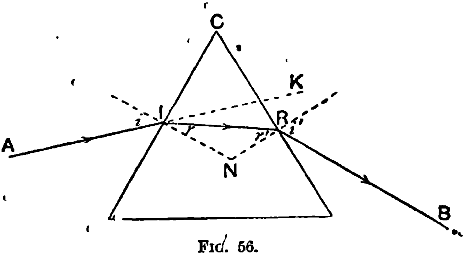


FIG. 56.

At R the ray is refracted again and passes out of the prism in the direction RB. If i' and r' are the angles between the normal NR and the ray outside and inside, the equation

$$\sin i' = \mu \sin r' \quad . \quad (2)$$

will hold for the refraction at emergence. In the triangle CIR the three angles must together be equal to two right angles, and hence the angles CIR and CRI must together be equal to

$$180° - a \quad . \quad . \quad . \quad . \quad . \quad (3)$$

but

$$CIR = 90° - r$$

and

$$CRI = 90° - r',$$

hence

$$CIR + CRI = 180° - (r + r') \quad . \quad . \quad . \quad (4)$$

By comparing (3) and (4) it is seen that

$$a = r + r' \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The three equations

$$\sin i = \mu \sin r \quad . \quad . \quad . \quad . \quad (1)$$

$$\sin i' = \mu \sin r' \quad . \quad . \quad . \quad . \quad (2)$$

$$a = r + r' \quad . \quad . \quad . \quad . \quad . \quad (5)$$

determine completely the path of a ray through a prism if its refracting angle a, and its index of refraction μ, are known. Thus if the first angle of incidence (i) is given, r may be calculated from (1), next r' may be calculated from (5), and finally i' from (2).

One case is of special importance, namely that in which the ray passes symmetrically through the prism as in Fig. 57. In this case $r = r'$ and from (1) and (2) $i = i'$; hence from (5) $r = a/2$.

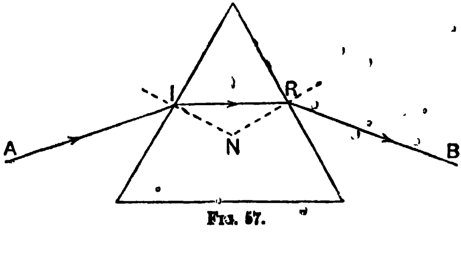


FIG. 57.

The *Deviation* of the ray by the prism is the angle through which the ray is rotated by the prism. If AI is the original direction of the ray (Fig. 56), and RB its direction after passing through the prism, the angle between AI and RB measures the deviation of the ray. We may obtain this angle by calculating the deviation of the ray due to the first, and that due to the second, refraction separately, and adding the results together. Thus the angle NIK is equal to i, the angle NIR is equal to r; and since the angle RIK is the deviation at the first refraction, this deviation is equal to $(i-r)$. Similarly at the second refraction the deviation is $(i'-r')$, and hence the whole deviation D is given by

$$D = (i - r) + (i' - r') = (i + i') - (r + r') = (i + i') - a \quad . \quad (6)$$

If i' has been found by equations (1), (2), (5), the deviation can be calculated. It is found that for a given incident ray, there is one position of the prism in which the deviation is less than in any other position; this is called the *position of minimum deviation*. It may be shown that the position of minimum deviation is the one in which the ray passes symmetrically through the prism (Fig. 57), that is to say when, as explained above,

$$i = i' \text{ and } r = \frac{a}{2} \quad . \quad . \quad . \quad . \quad (7)$$

the deviation given by (6) becomes in that case

$$D = 2i - a \quad . \quad . \quad .$$

or:

$$i = \frac{D + a}{2} \quad . \quad . \quad . \quad . \quad (8)$$

and as $\sin i = \mu \sin r$, we obtain by using (7) and (8)

$$\sin \frac{D + a}{2} = \mu \sin \frac{a}{2} \quad . \quad . \quad . \quad (9)$$

If D and a are measured, this equation will determine the refractive index μ, for

$$\mu = \frac{\sin \frac{D + a}{2}}{\sin \frac{a}{2}}$$

Numerical Example.—The angle of a prism is 60°, and its refractive index 1·6. Calculate the angle of incidence i when a ray of light passes with minimum deviation through the prism.[1]

EXERCISE

Geometrical Construction for the Emergent Ray

Let C be the refracting angle of the prism (Fig. 58). With centre C and radii 1 and μ, where μ is the refractive index of the material of the prism, draw two circles.

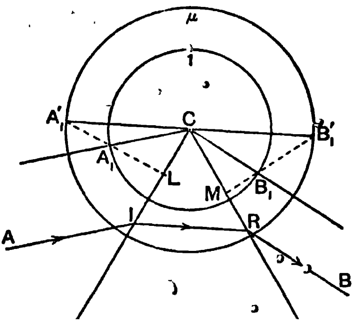


FIG. 58.

Let a ray parallel to A_1C be incident on the left-hand surface of the prism. From the point A_1 where A_1C

1 See table of sines, p. 247.

cuts the inner circle, draw A_1L perpendicular to the first face of the prism and produce this perpendicular till it meets the outer circle in A_1' Join $A_1'C$ and produce to again meet the outer circle in B_1'. Then, by the geometrical construction proved on pp. 123 and 124, $A_1'CB_1'$ is parallel to the refracted ray. Draw $B_1'M$ perpendicular to the second face of the prism, and let $B_1'M$ cut the inner circle in B_1. Join CB_1; CB_1 is parallel to the emergent ray.

Let AI be a ray parallel to A_1C, incident on the first surface at I. Draw IR through I parallel to $A_1'CB_1'$, cutting the second face in R. Through R draw RB parallel to CB_1. This line is the emergent ray corresponding to the incident ray AI.

Find, by this method the emergent rays corresponding to rays incident on the first surface of a prism of index 1·6 and angle 60°, at angles of 30°, 40°, 50°, and 60° respectively.

SECTION XXVIII

Exercise

Determination of the Refractive Index of a Glass Prism

Apparatus required.—Drawing board, prism, two lenses, slit, screen, boards, and protractor.

It will be necessary to obtain, in the first instance, a beam of light the rays of which are parallel. If a narrow slit is placed so that its centre is coincident with the principal focus of a lens, the rays coming from the centre of the slit will leave the lens parallel to the line joining the centre of slit and lens; the rays diverging from any other point of the slit will be approximately parallel among themselves, but inclined to the axis of the lens.

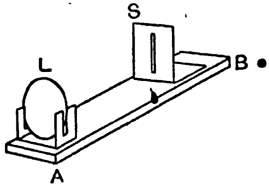


FIG. 59.

A small wooden board AB (Fig. 59), along which a lens L and a screen containing a vertical slit S can slide, is

provided. Place the lens near the end of the board, and find the position of the screen, such that the image of a distant object is seen sharply defined on it. If necessary, fasten lens and screen to the board by means of pins passing through the small holes in the corners of the supports. A combination of slit and lens which in this way produces a parallel beam of light, is called a collimator.

Place another lens L′ and a screen S′ down the middle of which a vertical line is drawn, on a similar board, so that the focus of the lens is in the centre of the screen. If necessary, attach the lens and screen to the board by pins. If this arrangement, which we shall call the focussing board,

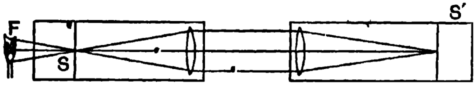


FIG. 60.

the collimator, and a luminous flame F (Fig. 60) are placed so that the centres of the screen, lens, slit and flame are in a straight line, a sharp image of the slit should appear on the screen.

EXERCISE.

In order to determine the refractive index, the angle of the prism has first to be measured.

Place the prism and focussing board in such positions (Fig. 61) that the beam of light is reflected from one of the faces AB, of the prism, and forms a sharp image of the slit along the centre line of the screen. Rotate the prism backwards and forwards a little to see that the image moves as it ought to do, and is not a spurious image due to *internal* reflections. When you are satisfied as to this, draw a line along one edge of the base on which the prism is mounted. Now turn the prism, without moving the focussing board, until a sharp image of the slit again appears in the same

position, the reflection now, however, taking place from AC instead of from AB, and draw another line along the same edge of the base. The angle between these two lines is that through which the prism has been rotated, which, it will be seen, is $180° - a$, if a is the angle of the prism.

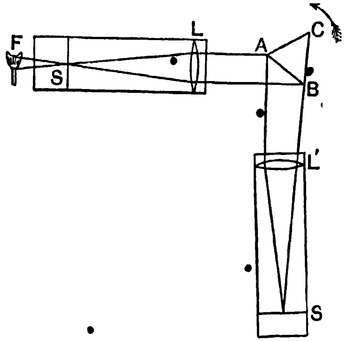


FIG. 61.

Proceed now to the measurement of the deviation produced by the prism, which is the second quantity necessary for calculating the refractive index.

Place the prism so that the light from the collimator is refracted through it, as in Fig. 61A; the angle which has

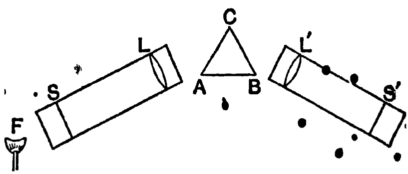


FIG. 61A.

been measured being the refracting angle C. Find the position of the focussing board, such that the spectrum produced by the prism appears on the screen. On turning the prism

round a vertical axis this spectrum will be found to change its place, but that position of the prism which gives the smallest deviation of the spectrum is easily obtained. With the prism in this position adjust the focussing board till the yellow of the spectrum falls on the central line of the screen, then mark the position of the board by a pencil line drawn along one of its edges.

Remove the prism and mark similarly the direction of the focussing board when it points directly to the collimator, as in Fig. 60, so that the image of the slit is seen on the central line of the screen.

The angle between these two lines on the drawing board is the angle of minimum deviation.

Repeat the adjustments and measurements and record as follows :—

Prism No. 6.

Angle of prism a	=	59°·5	60°
Angle of minimum deviation D	=	48°	48°
Sum $(D + a)$	=	107·5	108
$\frac{a}{2} = 29°{\cdot}7$ and 30° . $\sin\frac{a}{2}$	=	·494 [1]	·500
$\frac{D+a}{2} = 53{\cdot}7$ and 54° . $\sin\frac{D+a}{2}$	=	·806 [1]	·809

$$\mu = \frac{\sin\frac{D+a}{2}}{\sin\frac{a}{2}} = \frac{{\cdot}806}{{\cdot}494} = 1{\cdot}63 \text{ and } = \frac{{\cdot}809}{{\cdot}500} = 1{\cdot}62.$$

Mean value of $\mu = 1{\cdot}62$.

If a spectrometer is available, with a telescope which revolves over a graduated circular scale, and a prism table whose rotation can also be measured, the above observations should be repeated with the instrument, the angles of the prism and of minimum deviation being read to the higher degree of accuracy which the instrument permits.

[1] See table of sines, p. 247.

SECTION XXIX.

On Vision with the Naked Eye and through a Magnifying Glass

If an object or the image of an object formed by a lens, is looked at with the naked eye, its apparent size may be measured by the ratios of its linear dimensions to its distance from the eye.

For if in Fig. 62, AB is an object, ab the image formed on the retina, and if O is a point in the eye such that a

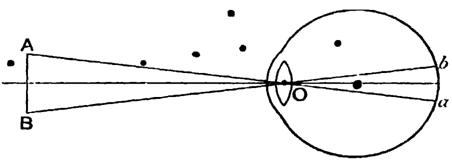


Fig. 62.

ray AO going towards O will continue along the same line Oa, we obtain in the similar triangles OAB and Oab the relation

$$\frac{ab}{Oa} = \frac{AB}{OA} \quad \text{or} \quad ab = \frac{AB}{OA}\,Oa.$$

The distance Oa is the same whatever the length or distance of AB, and hence the length ab of the image on the

retina is proportional to $\frac{AB}{OA}$ and this fraction we may call the apparent length of the line AB.

It is here assumed that there is in the eye a point having the property ascribed to the point O, and it can be shown that the assumption is justified. This point is situated in the crystalline lens near its back surface, and by means of it we may find the image of a luminous point A simply, by drawing a straight line AO and producing it to the retina.

When we speak of the distance of an object from the eye, we should, strictly speaking, measure it from the point O, but we shall commit no appreciable error by measuring it from the front surface of the cornea.

It is found by experience, that we can focus the eye so as to see distinctly objects lying within a certain range of distance.

The nearest point to which we can focus is called the "near point," and the farthest point is called the "far point" of the eye.

A normal or "emmetropic" eye has its far point at an infinite distance, and its near point 25 cms. from the eye.

Eyes which cannot focus for distant objects are called short-sighted or "brachymetropic," and the smaller the distance between the eye and the farthest point of distinct vision, the greater is the degree of short-sightedness. For such eyes the near point is generally nearer than 15 cms. Eyes having their near point farther than 30 cms. are called long-sighted or "hypermetropic," and their owners require spectacles to read or write with comfort. A long-sighted eye will often be able to focus rays which are convergent.

A common defect of the eye consists in one of its refracting surfaces not being accurately spherical but more curved in some directions than in others; the eye is then said to be astigmatic. When this defect is very

decided all luminous points will appear drawn out into a line. It is investigated by determining the nearest distance of distinct vision for each of a number of lines radiating from a point. For an astigmatic eye these distances are not the same.

The apparent size of an object depends on its distance from the eye. We shall therefore see a small object under the most favourable conditions if we place it as near the eye as possible, that is to say, at the nearest point of distinct vision.

If D_n is the nearest distance at which we can focus, and a the linear size of an object, a/D_n will be the greatest apparent size of the object, when seen distinctly with the naked eye.

We may now determine the advantage gained by looking at a small object through a convex lens placed close to the eye.

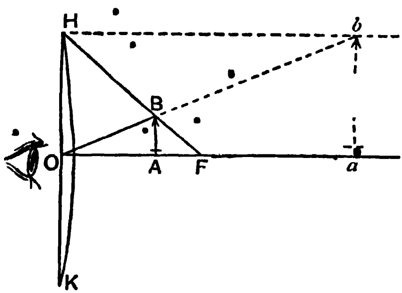


FIG. 63.

If AB is an object (Fig. 63) placed between a convex lens HK and its focus F, the rays BO and BH proceed after passing through the lens in the direction bO and bH, and appear to come from the virtual image b of B. ab is therefore the virtual image of AB, and $\frac{ab}{Oa}=\frac{AB}{OA}$ (Section XXIV.).

If the eye is close to the lens, Oa may be taken as the distance of the image from the eye, hence the apparent size of the image will be equal to $\frac{ab}{Oa}$, which is the same as the apparent size of the object would be if the lens were removed, the eye remaining in the same position. The advantage we gain by the lens lies therefore in the fact that we may, consistently with distinct vision, bring the object nearer to the eye than we can without a lens.

The magnification of the lens is defined as the ratio of the apparent size of an object as seen through the lens, to the apparent size when the object is looked at with the naked eye under the most favourable circumstances, that is to say, when at the nearest point of distinct vision. We have, therefore, the relation

$$m = \frac{AB}{OA} \div \frac{AB}{D_n} = \frac{D_n}{OA} = D_n\left(\frac{1}{OF} + \frac{1}{Oa}\right).$$

by (2) p. 129, Sect. XXIV.

Although we should obtain the greatest magnification by making the distance Oa as small as possible, *i.e.*, equal to D_n, in which case $m = \frac{D_n}{f} + 1$, and if D_n has its normal value 25 cms., is known as the "magnifying power" of the lens, it is better, when optical instruments are to be used for a considerable time, to adjust the distance of the virtual image so that it is as *far away* from the eye as possible, for in that case the muscles of the eye are at rest, and the eye suffers least fatigue.

If the eye is normal, so that it can focus for parallel rays, we may place the object AB at the focus, and OA becomes equal to OF the focal length of the lens. We then find the magnification $m = \frac{D_n}{f}$.

As D_n is always the same for the same eye, the magnification obtainable under these circumstances with different

lenses, is inversely proportional to the focal length of the lens. Hence $\frac{1}{f}$ is called the power of the lens. It will be seen from the above equation that no advantage is gained when the lens is used so as to fatigue the eye least, unless the focal length of the lens is less than the least distance of distinct vision; for if f is larger than D_n, m will be less than one, and the apparent size of the image as seen through the lens, will be less than the apparent size of the object as seen with the naked eye.

It will appear from the above that in order to obtain a magnification of 3 for instance, the nearest point of distinct vision being 21 cms., we should require to use a lens of 7 cms. focal length.

SECTION XXX

Determination of the Near and Far Points of the Eye, and the Magnifying Power of a Lens, and of a Telescope.

Apparatus required.—A mounted lens having a focal length of about 16 cms., three lenses mounted in contact having a combined focal length of between 2 and 4 cms., a focussing board, two small screens, and a telescope.

Exercise I

Determination of the Nearest and Farthest Points of Distinct Vision.

On one of the screens provided is a cross formed by two vertical lines, drawn with a sharp pencil as near together as possible, and two similar horizontal lines. Such a cross forms a convenient object for focussing.

Determine the focal length of the single lens provided with a screen with a small eye-hole, by focussing a distant object on the screen, and measuring the distance of the latter from the centre of the lens.

Now place the lens at one end of the focussing board (Fig. 64), and the screen at the other end. Bring the eye close up to the lens and slowly move the screen nearer to the lens until the small central square formed by the four lines of the cross is seen distinctly.

With a little practice and by moving the screen slowly, it will be possible to determine with considerable accuracy the point at which the eye first begins to see the cross distinctly. Measure the distance d_1 of the screen from the lens.

Place the eye once more against the lens and move the screen nearer and nearer until the cross *ceases* to look sharp and distinct. Measure again the distance d_2 between the screen and the lens.

Repeat both measurements three times with each eye.

Notice whether, by moving the screen farther or nearer than the limits of distinct vision, the horizontal lines remain sharp, while the vertical ones become indistinct or *vice versâ*. Record whether this is the case or not, and draw your own conclusions as to the astigmatism of your eyes.

In order that the measurements should be trustworthy the screen should be well illuminated. This is best secured by the student standing with his back obliquely towards a window or a gas flame so that the shadow of his head does not fall on to the screen. Record as follows:—

Focal length of lens observed $= f = 16 \cdot 5$ cms.

$\therefore$ Power of lens $= \frac{1}{f} = \cdot 0605$.

Eye.	d_1 cms.	d_2 cms.	$\frac{1}{d_1}$	$\frac{1}{d_2}$	$\frac{1}{D_f}$	$\frac{1}{D_n}$	Accommodation Power.	D_f cms.	D_n cms.
Right	12·0	7·7							
	11 6	7·8							
	12·0	7·7							
Means	11·9	7·7	·084	·130	·0235	·0695	·046	42·9	14·4
Left	12·5	7·7							
	12·3	7·8							
	12·4	7·8							
Means	12·4	7·8	·081	·128	·0205	·0675	·047	49·9	14·8

The calculations for the nearest and farthest points of distinct vision D_n and D_f are conducted as follows :—

The divergence of the beam incident on the lens from the screen at a distance d_1 is $1/d_1$, while that of the beam which enters the eye from the image at the farthest point of distinct vision is $1/D_f$. This decrease of divergence has been produced by the lens of focal length f, *i.e.* of power $1/f$, hence

$$\frac{1}{f} = \frac{1}{d_1} - \frac{1}{D_f} \quad \text{or} \quad \frac{1}{D_f} = \frac{1}{d_1} - \frac{1}{f}.$$

The calculation for D_n is carried on exactly in the same way.

If δ is the distance between the point O of the eye and the retina (Fig. 62), the *power* of the eye as a focussing instrument is $\frac{1}{D_n}+\frac{1}{\delta}$ or $\frac{1}{D_f}+\frac{1}{\delta}$ according as the eye is adjusted for its farthest or nearest point of distinct vision. The difference between these two quantities, *i.e.* $\frac{1}{D_n} - \frac{1}{D_f}$ may therefore be taken as a measure of the power of the eye to change its focal length. This is the power of accommodation of the eye, entered in the last column of the above table.

Exercise II

To Measure the Magnifying Power of a Lens

A board ABCD (Fig. 64) is provided, divided by a ledge EF, along which two small paper screens, H and K, can be moved. LL′ is a system of three lenses placed at the end of the board, so that its centre stands vertically over the centre line of the board. One of the screens, K, has two, and the other, H, a number of horizontal lines drawn on it 5 mms. apart. The latter screen serves as a scale on which the apparent distance between the

magnified images of the two lines drawn on the first screen is measured.

For this purpose it is necessary to look with one eye through the lens at the virtual image of the first screen, while the other observes the second screen directly.[1]

The left eye being shut, attention should be directed to the image of the screen K, seen through the lens with the right eye, and the screen should be placed as far forward as

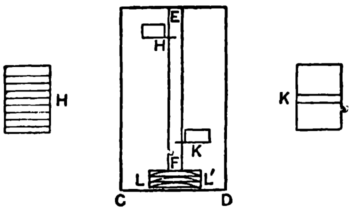


FIG. 64.

possible consistently with the two horizontal lines being seen distinctly. The image of K is then at the nearest distance of distinct vision for the right eye from the eye end of the board. Next, the second screen, H, should be placed at the further end of the board and observed with the left eye. It should then be moved towards the eye till both screens can be seen distinctly at the same time.

The magnified image of K will be seen to overlap the screen H which serves as scale. *The experiment consists in*

[1] The use of the two eyes is obviated by placing a small piece of mirror glass inclined at 45° so as to cover half the eye-hole of the lenses LL′. The screen H is then placed on an arm extending to the right in the direction CD. The right eye then sees the screen K through the lens and the screen H of reflection in the mirror, and the magnification observed is the *m* required in the subsequent calculations.

estimating how many divisions m' of the scale coincide with the apparent interval between the two lines drawn on the screen K. The measurement is facilitated by bringing the lower line of K into coincidence with one of the lines on H. This can generally be done by a slight backward or forward movement of H, or by a slight tilting of the head to one side. A fairly strong light should fall on the screens, and the estimate should be made to one-tenth of a division of the scale on H. The distance d of H from the eye end of the board should then be measured.

If the screen H had been placed at the nearest point of distinct vision D_n for the right eye, the apparent size of the division on H would have been larger than they appeared in the experiment in the ratio of d/D_n where d is the distance of H from the eye, and D_n the least distance of distinct vision for the right eye. Hence the magnifying power m, corresponding to the nearest point of distinct vision for the right eye, is

$$m = m' \times \frac{D_n}{d}.$$

But the magnifying power of the lens when so used has been shown (p. 154) to be given by the equation

$$m = \frac{D_n}{f} + 1.$$

Hence the focal length of the lenses may be calculated from the observations.

Record observations and results as follows:—

Distance d of screen H from eye . . .	19·5 cms.
Apparent size m' of interval on screen K seen through lens by right eye	7·6 divs. on H.
Least distance of distinct vision D_n for right eye	14·4 cms.

$$\therefore \text{ Magnifying power } m \text{ of lens for right eye } = 7{\cdot}6 \times \frac{14{\cdot}4}{19{\cdot}5} = 5{\cdot}6$$

$$\therefore m - 1 = 4{\cdot}6$$

$$\therefore \frac{1}{f} = \frac{m-1}{D_n} = \frac{4{\cdot}6}{14{\cdot}4} = {\cdot}32$$

$$\therefore \text{ Focal length of lenses} = \frac{1}{{\cdot}32} = 3{\cdot}1 \text{ cms.}$$

Similiar observations should be made with the eyes and screens reversed, and the value of f compared with that previously found.

The values may be checked by some method of measuring the focal length directly, but owing to the great thickness of the system used in these experiments, the ordinary methods described in this book are not applicable.

EXERCISE III

To Measure the Magnifying Power of a Telescope

Place the telescope at some distance from a scale or a wall on which the lines of division between the bricks can be seen distinctly. Focus the telescope till an image is seen in the eye-piece by one eye, while the other looks directly at the scale or wall. Move the head a little to one side and notice whether the two images of the wall move over each other or not. If they do, the focussing must be changed till the images remain fixed with respect to each other. When this is the case, count how many divisions seen direct are included in one division seen magnified through the telescope. This is the magnification of the instrument as used. If the distance of the telescope from the wall is changed, the magnification will be altered. It is necessary therefore to record this distance as well as the magnification.

PART V

SOUND

SECTION XXXI

The Sonometer

Apparatus required.—Sonometer, C and D tuning forks and small balance.

The object of this exercise is to verify experimentally the laws of transverse vibrations of strings or thin wires kept stretched in such a way that the tension remains constant during the vibration.

Definition.—The *frequency* of a vibration is the number of complete vibrations which take place in one second.

The equation

$$n = \frac{1}{2l}\sqrt{\frac{f}{m}}$$

gives the relation between the frequency of vibration n, the stretching force f, and the mass per unit length m of the wire.

The formula expresses a number of laws which may be tested experimentally.

EXERCISE I

If the stretching force remains constant, the number of vibrations per second varies inversely as the length of the string.

A steel pianoforte wire is stretched on the sonometer board provided (Fig. 65), and is furnished with a spring balance indicating the stretching force, and a bridge, which when placed under it at any point just supports it without altering the tension, thus enabling a shorter length of the string to be set into vibration. The upper edge of this bridge is provided with several notches at different

heights, and that particular notch should be used which gives the requisite support to the wire without increasing the tension.[1]

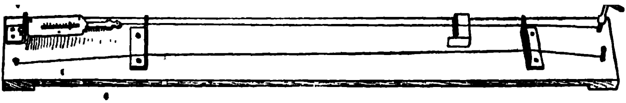


FIG. 65.

Find the lengths of wire which vibrate in unison with two tuning forks of known frequencies, and thus see how far the first law expressed by the above equation is verified by experiment.

The length of wire in unison with the tuning fork giving the lower note is first found as follows:—A light hoop-shaped "rider" of paper is wound round the middle of the sonometer wire, and the tuning fork is set into vibration. This is done by striking the end of the fork on the knee; it should not on any account be struck on the bench. The stretching force is then adjusted till the note given by the wire when vibrating is a little lower than that given by the fork. The fork is again struck on the knee, and the groove at the lower end of the handle of the fork is placed against the wire at one end, and the fork is slid slowly along the wire towards the middle, until a position is reached at which the rider is thrown into violent agitation. The wire will then be vibrating in tune with the fork. The bridge is then placed in the position occupied by the handle of the fork. Whether the bridge is correctly placed is then tested by placing the vibrating tuning fork on the bridge, or on the sonometer board under the wire, and noting the effect on the rider. If it

[1] A second wire is often provided which may be tuned to a desired note, and be used as the standard of reference instead of the fork.

is set into violent motion the bridge is correctly placed. If not, it should be moved one or two millimetres to the left or right, and the effect again tested. When the proper position has been found, the length of the wire vibrating should be measured.

The experiment should be performed three times and the mean taken, then observations should be made with the second fork and a second set with the first, and the results entered as in the following example:—

Fork.	Frequency.	Ratio.	Length of Wire.	Ratio
C	256		51·9 cms.	
D	288	1·125	46·1 ,,	$\frac{51\cdot8}{46\cdot1}=1\cdot124$
C	256		51·7 ,,	

$$\text{Difference} = \cdot001, \text{ or } \frac{\cdot001 \times 100}{1\cdot125} = \cdot1 \text{ per cent.}$$

Exercise II

For wires of the same length the frequency of the vibration varies directly as the square root of the stretching force

Tune the whole length of wire so that it is in unison with the D fork. This is best done by setting the fork in vibration, holding the handle down on the sonometer board, and varying the stretching force till the rider moves violently.

Observe the value of the stretching force. Then diminish the force till the wire is in unison with the C fork, and again read the balance. Make a second observation with the D fork. The observed ratio of the forces should be corrected for the error of the spring balance at

zero, which will be determined subsequently (see p. 169), and the results arranged as follows :—

Fork	Frequency.	Ratio.	Stretching Force in lbs.			$\sqrt{\text{Force}}$.	Ratio.
			Observed.	Zero.	Corrected.		
D	288		21·0	·5	20·5	4·53	
C	256	1·125	17·0		16·5	4·06	$\frac{4 \cdot 56}{4 \cdot 06} = 1 \cdot 123$
D	288		21·5		21·0	4·58	

Difference = ·002 = ·2 per cent.

Exercise III

For the same frequency, the length varies directly as the square root of the stretching force

Measure the length of wire vibrating in unison with the C fork as in the last experiment. Diminish the tension to about three-fourths of its former value, place the bridge under the wire, and find the length of wire which is now in unison with the C fork. Lower the stretching force again to about half of its original value, repeat the observations, and tabulate as follows :—

Stretching force in lbs.				$\sqrt{\text{Force}}$.	Ratios.	Lengths cms.	Ratios.
	Observed.	Zero.	Corrected.				
T	17·0	·5	16·5	4·06		55·7	
T_1	13·7		13·2	3·63	·89	50·2	·90
T_2	9·5		9·0	3·00	·74	41·8	·75

The differences are chiefly due to the uncertainty of the values of the stretching force.

Exercise IV

Determination of the frequency of vibration of a tuning fork

If the stretching force and the mass per unit length of the vibrating wire are known, the equation given above, p. 163, will allow us to calculate the frequency of the wire, and hence the frequency of a tuning fork which is in unison with the wire.

The stretching force is adjusted till the whole length of the wire is in unison with the C fork, and the stretching force is then read off on the spring balance. The adjustment is then repeated, the force again read and the mean taken. The length of wire vibrating is then measured, the tension in the wire relieved, the wire taken off, and at the marked places, the vibrating length of wire cut off with pliers and weighed, and the mass per unit length calculated from the length and total mass.

The sonometer board should now be held vertical, so that the spring balance is vertical and unloaded, and the reading for no load taken. In the example quoted it was found to be ·5, and this correction was applied to all readings of the stretching force before use was made of them.

Record as follows :—

Fork.	Vibration	Stretching force in lbs.			Length. cms.	Weight. grs.
		Observed mean.	Zero.	Corrected.		
C	256	17·0	·5	16·5	55·7	·50

Whenever a formula which establishes a numerical relationship between different quantities is used, it is necessary to express each quantity in the same system of units.

In the equation

$$n = \frac{1}{2l}\sqrt{\frac{f}{m}}$$

f, l, and m depend on the unit of length, and if l were measured in *centimetres*, and m taken to be the mass of the string per *foot* of length, a correct result would not be obtained. *Adopt the centimetre as the unit of length throughout.* The numerical value of f, the stretching force, depends on the unit of force which, if the gram is taken to be the unit of mass, is the *dyne*. In the instrument provided, this force is indicated in terms of the *pound weight*. In order to convert this into dynes, use is made of the fact that one pound is equal to 454 grams (see p. 25), and that the weight of one gram is equal to a force of 981 dynes. We find thus :—

$$t = 16{\cdot}5 \text{ lbs.} = 16{\cdot}5 \times 454 \times 981 = 7{,}350{,}000 \text{ dynes.}$$

$$m = \frac{{\cdot}50}{55{\cdot}7} = {\cdot}00896.$$

$$\therefore\ n = \frac{1}{2 \times 55{\cdot}7}\sqrt{\frac{7350000}{{\cdot}00896}} = \frac{1}{111{\cdot}4}\sqrt{820400000}$$
$$= 257.$$

For a standard C fork the number of vibrations per second is 256.

By taking two wires of the same material but of different radii, it may be verified experimentally that for the same stretching force the frequency varies inversely as the radius of the wire, *i.e.* as the square root of mass per unit length.

A wire of another material, *e.g.* brass may also be used to verify this statement.

SECTION XXXII

Resonance

Apparatus required.—Resonance tube and tuning forks.

If a tuning fork is held near the open end of a resonator, the resonator will resound if the note it can emit is the same as that of the fork. If the resonator consists of the air within a cylindrical tube closed at one end, the note it emits has a wave-length λ approximately four times the length l of the tube, *i.e.*

$$\lambda = 4l \text{ nearly} \quad . \quad . \quad . \quad . \quad (1)$$

or more accurately

$$\lambda = 4(l + \cdot 3d) \quad . \quad . \quad . \quad . \quad (1)'$$

where d is the inside diameter of the tube.

The relation between the velocity of sound V, the wave-length λ, and the frequency n, is given by the equation

$$V = n\lambda \quad . \quad . \quad . \quad . \quad (2)$$

If, therefore, two of the quantities—the frequency of a tuning fork, the velocity of sound in air, and the wave-length λ in air of the note emitted by the fork—are given, the third can be calculated from (2).

Combining the equations (1) and (2), we obtain the relation

$$V = 4nl,$$

or more accurately

$$V = 4n(l + \cdot 3d) \quad . \quad . \quad . \quad . \quad (3)$$

which can be tested experimentally if V and n are known.

The velocity V of sound in air varies with the temperature, according to the equation

$$V = 330\sqrt{1 + \cdot 0037t}$$

but may be calculated with sufficient accuracy (see p. 15) from the equation

$$V = 330 + \cdot 6t \quad . \quad . \quad . \quad . \quad . \quad (4)$$

when t is the temperature of the air in centigrade degrees, and V is measured in metres per second.

EXERCISE.

To determine the velocity of sound in air.

The two brass tubes provided telescope into each other (Fig. 66), so that the length of the column of air within the tube may be altered. The outer tube fits into a

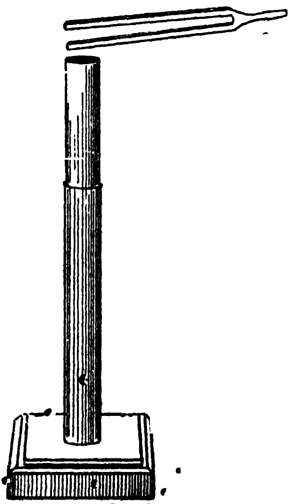


FIG. 66.

circular hole in a wooden base, and can be lifted away from the base. The exercise consists in adjusting the length l till the resonance is strongest, finding the velocity
rom the observation and comparing its value with that calculated from equation (4). The length of the tube should be altered and the adjustment and measurement repeated three or four times.

Record as follows :—

Temperature of air = 16°·4 C.
Fork C, 256 vibrations per second.

Observed length of resonating column of air	32·6 cms.
	32·3 ,,
	32·5 ,,
	32·6 ,,
Mean . .	32·5 ,,
Diameter d of tube 2·4 cm. .. $\cdot 3d =$	·7
Corrected length =	33·2 cms.

∴ Velocity of sound in air at 16·4° C. = 4 × 256 × 33·2
= 34100 cms. per sec.
= 341 metres per sec.

But by equation (4)

$$V = 330 + \cdot 6t \text{ metres per sec.}$$
$$= 330 + 9\cdot 8$$
$$= 339\cdot 8 \text{ metres per second.}$$

Difference = 1·2 metres = ·3 %.

Observations should be made in the same way with the D fork, and the results recorded as above.

Remove the tubes from the base and support them in the air so that both ends are open. Adjust to the length which resonated to the 256 C fork, and show that the tube when open at both ends resonated to the 512 C, *i.e.* the octave of the first note. Vary the length slightly till the resonance is most complete.

Verify the same fact for the octave of the D fork.

Record as in the above exercise, making the correction for each end of the table.

PART VI

MAGNETISM

SECTION XXXIII

Magnetisation

Apparatus required.—Magnetoscope, bar magnets, iron and steel wire.

The "magnetoscope" provided (Fig. 67) consists of a small piece of magnetised steel watch-spring, gummed to a strip of paper, and suspended in a glass bottle by means of a silk fibre. The use of the paper is to reduce or "damp" the oscillations of the magnet by the resistance the air offers to its motion.

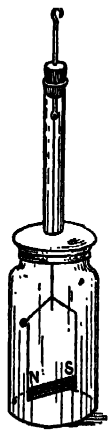


FIG. 67.

Bring a magnet near the instrument and notice that after its removal the magnetised spring returns to a definite position, such that the end marked N points a little west of north, thus behaving like a compass needle.

Show experimentally that one end of any magnet attracts one end of the needle and repels the other; and that this action is not diminished by introducing between the magnet and the needle, sheets of glass, wood, zinc, &c.

Bring a piece of steel watch-spring about 5 cms. long, which has been hardened by heating to redness and quenching in water, towards the magnetoscope. It has little or no effect on the needle. Lay the spring on the bench, and magnetise it by stroking

it once from the middle to the ends simultaneously, with two bar magnets, the one in the right hand having the end which attracts the N end of the needle downwards, the one in the left having the end which attracts the S end downwards.

Show that after one stroke the right hand end of the spring attracts the S and repels the N, and the left hand attracts the N and repels the S end of the needle of the magnetoscope.

Repeat the stroking, and show that these attractions and repulsions increase with each stroke up to a certain point, when the spring is magnetised as strongly as it can be with the magnets used.

Suspend the spring by means of a fine silk fibre, and observe that it sets itself in the same direction as the needle of the magnetoscope.

Calling the ends which point to the north the N ends, those which point to the south the S ends, show that like ends of the needle and watch-spring repel, while unlike ends attract, each other.

Take a piece of soft iron wire, about ·5 mm. diam., and about 5 cms. long, which has been well annealed by being heated to redness and allowed to cool slowly, stroke it with the magnets as before, and show that it acts only feebly on the needle. Lay it, pointing east and west, on the bench, at some distance from any magnets, and tap it sharply several times with a pencil. Show that it has now lost its magnetism.

Take a short bar magnet, a little longer than the needle of the magnetoscope, determine which end of it attracts the N end of the needle, and lay it alongside and parallel to the needle, with that end to the north. Bring one end of the soft iron wire nearly into contact with one end of the magnet, and the other end as near as possible to the corresponding end of the needle. That end of the needle is strongly attracted, and this will still be the case if the

wire is reversed, thus showing that the end of the wire farther from the pole of the magnet, is magnetised in the same way as that pole.

Remove the magnet, and show that the wire is only feebly magnetised.

Repeat the experiment, this time bringing the end of the wire into contact with the end of the magnet and notice that the effect is increased.

This action of a magnet on a piece of iron near it, whereby the iron behaves for the time being as a magnet, is called *Magnetic Induction.*

Replace the iron wire of the above experiments, by a hardened steel spring, and notice that the action of the steel on the magnetoscope needle is much less intense, but that the steel continues its effect when the magnet is removed. The iron in the presence of the magnet is more "susceptible" of being magnetised than the steel, but the steel is more "retentive" of its magnetism than the iron.

The magnetic behaviour of a bar of iron or steel may be imitated by means of a closed test tube nearly filled with iron filings. Place a magnet at each end of the tube so that tube and magnets are in one straight line, and the magnets have unlike ends towards each other. Rotate the tube about its axis. The iron filings become magnetised, and set themselves with their greatest lengths along the line joining the poles of the two magnets. The tube therefore acts as a magnet, and this action continues so long as the filings are not shaken. On shaking, the action ceases. Test these statements by experiment.

Show that a piece of iron wire, if tapped or bent in the neighbourhood of a magnet, becomes magnetised.

The fact that magnetic needles point to the north, indicates that the earth itself is a large magnet, having its N attracting parts in the northern hemisphere, and if this is the case it ought to be possible owing to the

magnetic effect of the earth, to magnetise a piece of iron by simply tapping or bending it.

Take a piece of unmagnetised iron wire, place it on the bench with its length north and south, and tap it or bend it. It will be found to be a weak magnet. The same is found if the wire is held vertical, but not if it is placed horizontal with its length east and west.

Dip the piece of magnetised watch-spring into iron filings, and notice that the filings adhere to the ends only, and not to the middle. Break the spring into two pieces, and show that filings will adhere to the ends of each piece. Place the two pieces together again on a piece of gummed paper, and show that the middle no longer takes up filings. Explain what conclusion as to the constitution of a magnet could be drawn from these observations.

A body which behaves like iron with respect to magnetism is called a "magnetic" or "paramagnetic" body. Cobalt and nickel are paramagnetic bodies.

The student should record in his note book the experiments made and their results, giving diagrams of the apparatus used.

SECTION XXXIV

Magnetic Forces

Apparatus required.—Bar magnets, small compass, and muslin bag with iron filings.

It is evident from the preceding experiments, that a magnetic body in the neighbourhood of a magnet is acted on by forces due to that magnet, and that a small compass needle placed at any point will set itself in the direction of the force at that point.

It has been proved by experiment, that the action of a magnet on a small compass needle, is nearly the same as that of two magnetic masses of opposite kinds, situated near the ends of the magnet.

If the magnet is in the form of a long thin wire, the magnetic forces appear to emanate from two points known as "poles" near the ends of the needle. A magnet for which this were strictly true would be called a "simple magnet." No magnet we meet with in nature is a simple magnet, and the expression "pole," when it refers to ordinary bar magnets, is used to indicate the *region* of the magnet, from which "lines of force" (see Exercise III.) seem to diverge.

EXERCISE I

To find in a long bar magnet the positions of the regions which may be considered the Poles

Place the magnet in the middle of a sheet of paper on a drawing board. Draw a pencil line round the magnet to mark its exact position, and remove it.

Place the compass needle on the board and notice the direction in which it points when *no magnets are near it.* A vertical plane through the needle is called the "magnetic meridian," and the direction in which the head of the needle points "magnetic north."

Replace the magnet and place the compass about a cm. from one end of it (Fig. 68). Rotate the drawing board till

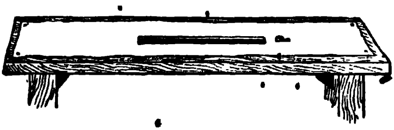


FIG. 68.

the needle is in the magnetic meridian, with its pointed end to the north. Mark on the paper close to the compass box, the direction in which the needle points, and after removing the compass, draw in that direction a straight line through the point occupied by the centre of the needle. This straight line will be found to cut the geometrical axis of the magnet, at a point about $\frac{1}{12}$ of the length of the magnet from the end. Repeat these observations at eight different positions round one end of the magnet. The eight lines thus determined meet in a region which may be called a "pole" of the magnet.

Repeat the observations at the other end of the magnet to obtain the position of the other pole.

Measure the distance between the poles and find for the magnet used, the ratio of the distance between the poles, to the total length of the magnet.

Record as follows :—

Length of magnet	12·0 cms.
Distance between poles . . .	10·1 „

$$\text{Ratio} = \frac{10{\cdot}1}{12} = {\cdot}84$$

Exercise II

To find the direction of the resultant Action of two equal and opposite magnetic Poles on a small magnetic needle placed at any point

It is explained in text books, that if a magnetic pole could be isolated from its accompanying pole of opposite kind, it would act on a similar isolated pole with a repulsive force inversely proportional to the square of the distance of the poles apart. If the poles are of unlike kinds, the force is an attraction following the same law. Hence, to find the direction of the resultant force on an isolated pole at any point, due to the two poles of a magnet, we must compound the two forces, one an attraction and the other a repulsion, due to the two poles. Let A and B be the poles of the magnet in the previous experiment, and let the direction of the resultant force be required at a point P (Fig. 69). Join PA, PB and measure

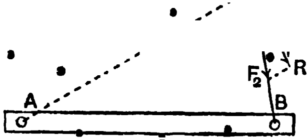


Fig. 69.

the lengths AP, BP. Suppose the isolated pole at P to be of the same kind as A. Then the force at P due to A is a repulsion, and that due to B is an attraction. Produce AP to F_1, and in BP take F_2, such that PF_1 and PF_2 are proportional to $1/AP^2$ and $1/BP^2$ respectively. Complete the parallelogram of which PF_1 and PF_2 are intersecting sides. Let the fourth corner be R. Then PR is the direction of the resultant force at P, due to the action of the two poles A, B.

Determine in this way the directions of the resultant forces at four points in a line parallel to AB.

Put the magnet again in position, and place the small compass with its centre over one of the points P. Then if there were no magnetic forces acting save those due to the poles A and B, the needle ought to set itself along the line PR. As the influence of the earth on the compass is not very small compared to that of the magnet, the drawing board should be turned in making this experiment, so that the compass needle when at P points to the north, as the effect of the earth in disturbing the experiment is then least. Even when this is done, it will be found that the compass does not set itself accurately along PR, and it will thus be seen that the action of a magnet is not identical with that of two poles alone.

Mark on the paper the direction in which the needle sets.

Determine experimentally in this way the direction of the forces due to the whole magnet at the four points selected.

Draw to scale in your note-book a diagram showing the positions of the four points with respect to the magnet, and indicate the directions of the forces due to the poles alone by dotted lines, those due to the entire magnet by full lines.

By taking a large number of points, the directions of the resultant force due to the poles alone could be found all over the paper, and be compared with those found

as above for the magnet itself, but the method of determination would be too long. By making use of the fact that iron filings, under the influence of the entire magnet, set themselves with their lengths along the direction of the resultant force, we can readily determine these directions, and thus see the nature of the "field of force" due to the magnet.

EXERCISE III

To find the Lines of Force due to magnets in various positions

Take a short bar magnet, lay it on the bench, and alongside it place slabs of wood of the same thickness as the magnet. Over the magnet and slabs lay a piece of paraffined paper, and secure it by weights. Shake from the muslin bag a few iron filings on to the paper, tap it gently with a pencil, and notice that the filings set themselves along certain lines, which are "lines of force." When the curves are distinct pass over the paper a Bunsen flame to melt the paraffin and fix the filings. Compare the curves thus obtained with the drawing of the lines of force due to two poles, given in the text books.

Determine in the same way the lines of force between two magnets placed in the same straight line, first with unlike poles about 5 cms. apart, then with like poles 5 cms. apart. Also determine the lines between one end of a bar magnet and a bar of soft iron 3 cms. from the end, with its axis perpendicular to that of the magnet.

Reduced diagrams of the lines of force in each case should be copied into the note-book.

Examine the curves obtained carefully, and endeavour to show from them, that magnetic actions may be explained by supposing tensions acting along the lines of force (*i.e.* supposing lines of force to be like elastic bands), and pressures at right angles to them.

SECTION XXXV

Magnetic Survey of the Laboratory

Apparatus required.—Magnetometer.

If a compass needle is suspended so that it can move freely in a horizontal plane above a vertical axis, it comes to rest in the magnetic meridian, *i.e.* in the direction of the lines of horizontal magnetic force at the point of suspension. By placing the needle at different points in the laboratory, it is possible to determine whether the lines of magnetic force are all parallel to each other. It is generally found that this is not the case, owing to the presence of iron in the walls, floors, etc., of the room, and it becomes of importance to know exactly the *direction* of the lines throughout the room. Since however the records of most current-measuring instruments depend on the *magnitude* of the earth's horizontal magnetic force, it is advisable to determine at a number of points in the room, both the direction and magnitude of this force. The direction is determined readily as above, and the relative magnitudes of the forces at different points may be found, by allowing the magnetic needle to oscillate in a horizontal plane about the vertical axis of suspension, observing the times of oscillation T, and using the fact that

$$T = 2\pi \sqrt{\frac{I}{MH}},$$

where I is the moment of inertia of the suspended system about the axis of suspension, M is the magnetic moment of the magnet, and H is the horizontal component of the earth's magnetic force. Thus if I and M are kept constant, the times of oscillation will vary inversely as the square roots of the magnetic fields, or the fields will vary inversely as the squares of the time of oscillation.

EXERCISE

The circular box provided contains a magnetised needle, which is suspended by a fibre so that it can oscillate

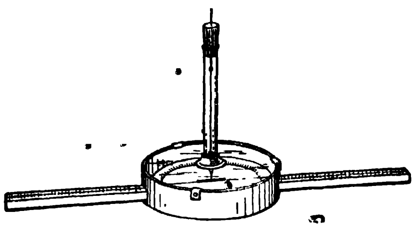


FIG. 70.

above a graduated circular scale placed in the bottom of the box (Fig. 70). To the needle a long thin pointer is attached, both ends of which fall over the graduated scale. Readings of each end of the pointer are taken at each observation, and the mean taken in order to eliminate any error which might be introduced owing to the needle not rotating about an axis through the centre of the scale. The zeros of this scale are placed in a direction perpendicular to that of the projecting centimetre scales. Place the box so that these scales are perpendicular to that wall of the laboratory which runs most nearly north and south. Raise the fibre by means of the wire at the top of the tube till the needle

swings freely over the scale, and after observing a few swings lower the fibre when the needle is near the centre of a swing, then raise it again. Repeat this raising and lowering till the swing is entirely stopped, then raise the needle and read both ends, noting whether the north end is east or west of the zero of the scale, and take the mean of the readings on both sides.

Set the needle oscillating over an arc not exceeding 45° and count the time of five or ten complete oscillations. Repeat the observation three times and take the mean.

Determine in this way the directions and relative magnitudes of the magnetic force at the points marked 1, 2, 3, 4, on the plan of the laboratory on the laboratory notice board.

Arrange the results as follows, drawing also a plan of the laboratory in your note-book.

Position.	Direction.	Time of oscillation.	$\frac{1}{T^2}$
1	10° E	7·75 seconds	·0166
2	11° E	8 25 ,,	·0147
3	19° E	7·8 ,,	·0164
4	3° E	7·75 ,,	·0166

From this table it is seen that there are disturbing causes at work near positions 2 and 4. In order to investigate their nature, observations should be made at points near these positions, and sufficiently close together to enable the lines of force to be drawn from the results. It will generally be found that the irregularities are thus traced to iron gas or water pipes, or beams in the fabric or fittings of the laboratory.

SECTION XXXVI.

Determination of the Magnetic Moment of a Magnet and the Intensity of a Magnetic Field

Apparatus required.—Magnetometer, magnet, and vibration box.

The determination involves two experiments, one the "vibration experiment," which gives the product, the other the "deflection experiment," which gives the quotient of the two quantities required.

EXERCISE I

Vibration Experiment.

Suspend the magnet horizontally, in the box provided (Fig. 71), by means of a fine silk fibre, and see that it can oscillate freely in a horizontal plane.

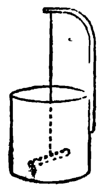


FIG. 71.

Place the box at one of the positions marked on the plan of the laboratory, deflect the magnet about 20° from its position of equilibrium by means of another magnet, determine the number of seconds the magnet takes to perform ten complete oscillations, and divide by 10 to get the time of one oscillation. Take three observations and let T be the mean time of one oscillation.

The value of T is connected with the dimensions, mass, and magnetic moment of the magnet, and the intensity H of the earth's horizontal field, by the equation.

$$T = 2\pi \sqrt{\frac{I}{MH}}$$

where M is the magnetic moment, and I depends solely on the shape and mass, and is called the moment of inertia, of the magnet. In the case of a rectangular bar if m is the

mass, $2a$ the length, and $2b$ the breadth, the moment of inertia is given by the equation :—

$$I = m\frac{a^2 + b^2}{3}.$$

The magnetic moment of a simple magnet, having two points as poles, is equal to the product of the strength of each pole and the distance between them. In the case of bar magnets we must content ourselves with the experimental determination of the magnetic moments.

From the above equations we deduce

$$MH = \frac{4\pi^2 I}{T^2} = \frac{4\pi^2}{T^2} \cdot m\frac{a^2 + b^2}{3}$$

in which M and H are unknown, T has been found by the preceding observations, m, a, b can be found by weighing and measuring the magnet.[1]

Exercise II

Deflection Experiment

Adjust the magnetometer provided so that the needle oscillates freely above the graduated circle. Rotate the box till one end of the needle comes to rest over a zero division. The other end of the needle will then be either over or not far from the other zero division. The line joining the zeros will then be in the magnetic meridian, and the projecting centimetre scales on the box will point magnetic east and west, with sufficient accuracy for the present purpose.

Read both ends of the needle, noting whether the readings are east or west of the zeros.

Place the bar magnet, of which the time of oscillation has just been found, on the scale projecting to the west, with its north end pointing towards the circular box. The magnetometer needle will be deflected out of the magnetic meridian. Bring it to rest by lowering and raising the fibre. Vary the position of the magnet till the deflection is about 45°, then take readings

[1] The equation may also be used as a basis for the comparison of the magnetic moments of magnets allowed to oscillate in the same place and their times of oscillation observed.

of each end of the magnet and of the magnetometer needle, noting in the case of the latter whether the readings are east or west of the zeros.

Now reverse the magnet on the scale, placing it at the same scale readings. The direction of deflection of the needle will be reversed. Read the deflection.

Remove the magnet from the west, and place it at the same distance to the east of the box.

Take readings, reverse the magnet, and again take readings.

Find the reading of the centre of the magnet on each scale by taking the means of the end readings. With a beam compass measure the distance apart of these central readings. This is twice the mean distance d of the centre of the magnet from the needle.

Find by the table (p. 241) the tangents of the angles of deflection observed, and take the mean.

If M is the magnetic moment of the magnet, $2l$ the distance between its poles, which may be assumed for the purpose of the present exercise to be five-sixths of the length of the magnet, H the horizontal intensity of the earth's magnetic force, and $\tan\theta$ the above mean,

$$\frac{M}{H} = \frac{(d^2 - l^2)^2}{2d} \tan\theta.$$

From the value of M/H thus determined, and the value of MH previously found, the values of both M and H can be calculated.

Arrange observations and results as follows :—

Vibration Experiment.

Magnet No. 4. Position 2 in laboratory.

Length of magnet	= 6 cms.	$a = 3$	and	$9{\cdot}0$
Breadth ,, ,,	= 0·48 ,,	$b = {\cdot}24$	$b^2 =$	$\cdot 06$
			$a^2 + b^2 =$	$9{\cdot}06$
			$\frac{1}{3}(a^2 + b^2) =$	$3{\cdot}02$
Mass	= 11·06 grams.		$\frac{m}{3}(a^2 + b^2) =$	$33{\cdot}4$

Times of oscillation = 7·05, 7·0, 6·95. Mean = 7·0 seconds.

$$\frac{4\pi^2}{T^2} = \frac{4 \times 9{\cdot}87}{49} = \frac{39{\cdot}48}{49} \doteq {\cdot}806$$

$$\therefore \quad MH = 33{\cdot}4 \times {\cdot}806 = 26{\cdot}8.$$

Deflection Experiment

Magnetometer No. 4. Position 2 in laboratory.

Position of Magnet No. 4.			Pointer Readings.		Deflections.			Tangents.	Mean tan θ
Ends.		Middle	North.	South	North	South	Mean		
N.	S.								
		Zero	0	$2\overset{\circ}{\cdot}5$ W.					
11·0	17·0	14·0 West	$43\overset{\circ}{\cdot}2$ E.	45·4 W	$43\overset{\circ}{\cdot}2$	$42\overset{\circ}{\cdot}9$	$43\overset{\circ}{\cdot}0$	·93	
17·0	11·0	14·0 ,,	43·0 W.	40·4 E.	43·0	42·9	42·9	·93	925
17·0	11·0	14·0 East	42·8 E.	45·2 W.	42·8	42·8	42·8	·92	
11·0	17·0	14·0 ,,	42·7 W.	40·2 E.	42·7	42·7	42·7	·92	
		Zero	0	2·5 W.					

Distance $2d$ from 14·0 West to 14·0 East = 28·04 cms.

$$\therefore \quad d = 14{\cdot}02 \text{ cms.} \qquad \therefore \quad d^2 = 196$$

$$l = \tfrac{5}{6}(3) = 2{\cdot}5 \qquad \therefore \quad l^2 = 6$$

$$d^2 - l^2 = 190$$

$$\frac{M}{H} = \frac{(190)^2}{28{\cdot}04} \times {\cdot}925 \qquad = 1193$$

Hence

$$H^2 = \frac{MH}{\frac{M}{H}} = \frac{26{\cdot}8}{1193} = {\cdot}0225.$$

$$\therefore \quad H \text{ at position } 2 = {\cdot}150$$

$$\text{and } M^2 = MH.\frac{M}{H} = 26{\cdot}8 \times 1193 = 31970.$$

$$\therefore \quad M \text{ of magnet No. } 4 = 179.$$

Magnetic moment per gram of magnet No. 4 = 179/11·06 = 16·3.

PART VII

ELECTRIC CURRENTS

SECTION XXXVII

EXERCISE

Action of Currents on Magnets

Apparatus required.—Simple cell, compass box, connecting wire, and connectors.

Two plates of copper and zinc and a jar are provided (Fig. 72). Half fill the jar with very dilute sulphuric

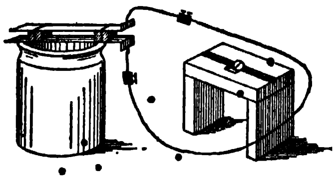


FIG. 72.

acid and insert the plates, taking care that the acid does not touch the upper part of the plates where the wires are attached, and that the plates do not touch each other. Connect the terminals of the cell thus formed, by means of "connectors," to the ends of a piece of thin copper wire about a metre long.

(*a*) *Single Horizontal Current.*

Place the small compass on the table, taking care that the wire from the cell does not pass near it. Rotate the compass box till the pointed end of the needle comes over the mark N of the dial. Arrange the wire from the cell so that about one third of its length lies in the magnetic meridian above the needle, and the rest of it is at some distance from the compass. Now bring down the length which is parallel to the needle, towards the needle, and

notice that the needle is deflected. Note the position taken up by the needle, when the wire touches the glass of the compass box. Reverse the wire, so that the current now passes in the opposite direction over the compass box. Notice, that on bringing down the wire, the needle is deflected by an equal amount in the opposite direction to what it was in the previous experiment.

Raise the plates out of the liquid.

When a current passes through a wire, it creates in the neighbourhood of the wire a magnetic field, the lines of force of which are circles with their centres in the wire and their planes perpendicular to it. The direction in which a north magnetic pole would be moved along these circles, if it were placed in the magnetic field, is related to the direction of the current in such a way, that if we suppose a righted-handed screw (*e.g.* a corkscrew) to be driven along the path of the current, the direction in which the screw is rotated is that in which the pole would be moved, *i.e.* is that of the magnetic force.

Using this rule, determine from the observations the direction in which the current in the wire flows, and hence which terminal of the cell is the positive one.

Place the wire in a groove running magnetic north and south in a piece of wood and the compass over the groove, so that the north end of the needle is directly over the N mark of the dial. Now insert the plates in the liquid and notice that the deflection is equal and opposite to that obtained when the current passed in the same direction above the needle. Raise the plates, reverse the direction of the iron in the groove, insert the plates and observe that the deflection is reversed.

Raise the plates, place the wire east or west of, and as close as possible to, the compass box, keeping it parallel to the magnetic meridian, and in the horizontal plane through the needle. Notice that on lowering the plates the current now appears to have no effect on the needle.

Record these observations, and show that they are all in agreement with the above rule.

(*b*) *Single Vertical Current.*

Pass the wire through the vertical hole in the stand provided, and arrange it so that some length of it, both above and below the hole, is vertical. Place the compass needle so that its centre is close to, and west of the wire, and its north end over the N mark of the dial. Lower the plates into the liquid and observe the position of the needle. Repeat the observations with the centre of the needle, east, north, and south, respectively, of the wire, showing that where deflections of the needle are observed, the deflections are in the opposite direction when the direction of the current through the wire is reversed by reversing its connections to the cell.

Show that the observations can be explained by the fact that the earth's magnetic force tends to set the needle in the magnetic meridian, and that it is the resultant of this force and the force due to the current, which determines the direction of the needle. Draw diagrams, showing in each case, (1) the direction of the force on a north pole due to the current, (2) the direction of the force due to the earth, (3) the resultant force on the supposition that the magnetic force due to the current is equal to that due to the earth.

(*c*) *Multiple Currents.*

Place the wire again in the horizontal groove and the compass over it, and determine the deflection of the needle due to the current in the wire on inserting the plates in the liquid, when the other parts of the circuit are at some distance. Now double the wire so that a length of it passes back over the compass. Notice that the deflection of the needle is increased, and that by arranging the wire so that it passes twice under and twice over the needle,

the effect is again increased. By winding a wire a great number of times in this way about a magnetic needle, an extremely small current can be detected.

Show that when a wire is bent sharply back on itself the effect of a current flowing in it on the needle is small, even when it is brought near the needle, so long as the two parts of the bent wire are at nearly the same distances from the needle.

Hence when a wire carrying a current in the neighbourhood of a magnetic instrument is to produce as little effect as possible on the instrument, the wire should be arranged so that near the instrument it is folded back on itself, so that the current in one part flows in one direction and in the other in the opposite. The unit in which electric currents are measured is the "Ampère." Its value is such that the intensity of the magnetic field produced by a unit current flowing through a wire bent into the form of a circle of radius r is, at the centre of the circle, equal to $\pi/5r$.

(*d*) *Galvanometers.*

An instrument in which the action of a current on a magnetic needle is used to measure the strength of the current, is called a galvanometer.

It can be shown that if the wire of a galvanometer is sufficiently far away from the magnetic needle, and the instrument is placed with the plane of the wire in the magnetic meridian, the current required to produce a given deflection of the needle, is proportional to the tangent of the angle of deflection. A galvanometer which satisfies this condition is called a "tangent galvanometer." It serves to compare the strengths of electric currents, and if the radius of the circle formed by the wire and the number of turns are known, the current may be measured in "Amperes."

SECTION XXXVIII

The Voltaic Cell and Tangent Galvanometer

Apparatus required.—Two Leclanché cells, tangent galvanometer, resistance coils, plug key, and connecting wires.

Each voltaic cell possesses a certain power of driving an electric current against the resistance offered by a circuit. This power, known as the Electromotive Force of the cell, depends on the materials composing it, that is on the nature of the liquid and of the plates, but not on the shape of the plates or their positions in the liquid. Thus, two cells composed of plates of copper and zinc in dilute sulphuric acid would have the same Electromotive Force, although one cell might have plates twice as large as the other, or twice the distance apart.

The unit in which electromotive forces are measured is called a "Volt."

The current which a cell can send through a circuit, depends on the Electromotive Force driving the current, and on the Resistance which the circuit opposes to the passage of the current. This Resistance is the sum of two, that offered by the cell itself, and that offered by the circuit outside the cell.

By Ohm's Law, the current generated is equal to the

Electromotive Force in the circuit, divided by the resistance of the circuit. If we call E the Electromotive Force of the cell, B its resistance, called the "internal resistance," R the resistance of the rest of the circuit or the "external resistance," and C the current generated, then

$$C = E/(B + R).$$

If the cell were of the simple kind used in the experiments in the previous paper, B would be nearly proportional to the distance of the plates apart, so that by decreasing this distance, the current which the cell would send through any circuit of which it formed a part could be increased. B would also be nearly inversely proportional to the area of the plates, so that by increasing the size of the plates the current in the circuit would again be increased.

EXERCISE

To Determine the Internal Resistance of a Cell

The coil of the small tangent galvanometer provided (Fig. 73) consists of three or four turns of wire. The

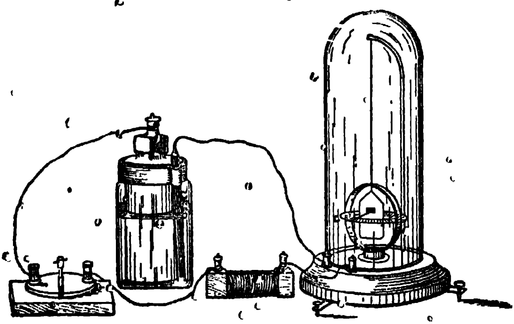


FIG. 73.

needle is suspended in the middle of the coil by a long silk fibre. To the needle are attached a paper "damper," and a pointer at right angles to the needle. The angles through which the pointer is deflected are read off on a scale underneath it graduated in degrees.

Place the galvanometer on the bench in such a position that the needle is in the plane of the coils, and the pointer therefore at right angles to that plane. Adjust the levelling screws till the needle rotates freely. Read the positions of the ends of the pointer, which should not differ greatly from zero. If they do, note whether they are north or south of the zeros.

Call these the zero readings.

The Leclanché cell provided is composed of a zinc and of a carbon rod dipping into saturated solution of ammonium chloride. Connect one terminal of the cell through the plug key to one terminal of the resistance coil marked "2 ohms" provided. Connect the other terminal of the cell, and the other terminal of the resistance coil, to the terminals of the galvanometer. When binding screws are not provided on the cell, use "connectors."

When the pointer has come to rest take readings of each end, noting whether a reading is north or south of the zero.

Interchange the connections at the cell, so that the current now passes through the galvanometer in the opposite direction, and take readings again. Now disconnect the resistance coil from the wires and connect the free ends of the wires together. Again take readings. Reverse the cell once more and take readings.

Repeat the observations with the coil in circuit.

Disconnect the cell, and take readings of the zero of the galvanometer. These ought to coincide with the previous readings.

Record and reduce the observations as follows :—

Galvanometer No. 3. Cell No. 1. Coil No. 4.

Experiment.	Readings.		Angles of Deflection.				Tangent.[1]	Mean with coil
	East.	West.	East.	West.	Mean.	Mean.		
Zero	0°	2° S.						
With coil . . .	47° S.	46 N.	47°	48°	47°·5	48°·0	1·11	
	48 N.	51 S.	48	49	48·5			
Without coil . . .	66 N.	67 S.	66	65	65·5	65·5	2·19	1·09
	65 S.	64 N	65	66	65·5			
With coil . . .	46 S.	45 N.	46	47	46·5	47·0	1·07	
	47 N.	50 S.	47	48	47·5			
Zero	0	2 S.						

If R is the resistance of the coil, B that of the cell, connecting wires and galvanometer, in the experiment with the coil in circuit we have :—

$$\text{Current with coil in circuit} = \frac{E}{B+R}.$$

In the experiment without the coil we have similarly :—

$$\text{Current without coil} = \frac{E}{B}.$$

Dividing the second of these equations by the first we get—

$$\frac{\text{Current without coil}}{\text{Current with coil}} = \frac{B+R}{B} = 1 + \frac{R}{B}.$$

The ratio of the currents observed is equal to that of the tangents of the angles of deflection, hence, substituting the values of the tangents from the table, we have :—

[1] See table of tangents, p. 247.

$$\frac{2 \cdot 19}{1 \cdot 09} = 1 + \frac{R}{B}$$

$$\therefore \frac{R}{B} = \frac{1 \cdot 10}{1 \cdot 09}$$

$$\therefore B = R\frac{1 \cdot 09}{1 \cdot 10} = \cdot 99R.$$

Hence, the value of B can be found if that of R is known.

The unit in which resistance is measured is the "Ohm," and the coil R has a resistance of 2 ohms. Hence B of cell No. 1, connecting wires and galvanometer = 1·98 ohms.

As the resistance of the galvanometer and connecting wires is extremely small compared to that of the cell, we may take B to be the resistance of the cell.

Repeat the above observations with another cell.

Next connect the two cells in series, *i.e.* connect the carbon of one cell to the zinc of the other, and determine the resistance of the combination, using as external resistance two of the 2 ohm coils connected in series.

The resistance of the cells in series should equal the sum of the resistances of the two cells.

Now connect the two cells in parallel, *i.e.*, connect the two zincs together and the two carbons together, attaching one of the wires from the galvanometer and resistance to the zincs, and the other to the carbons, and determine the resistance of the combination. In this experiment use the 1 ohm coil provided.

The resistance of the cells in parallel is about half that of each cell.

NUMERICAL EXERCISE.—Find, by calculating the current in each case, which is the better way to connect two similar cells, each having a resistance of 2 ohms, in order to get the greater current, when the resistance in the external circuit is 1, 2, 4 ohms respectively.

SECTION XXXIX

Measurement of Resistances by the Resistance Bridge

Apparatus required.—Resistance bridge, astatic galvanometer, Leclanché cell, plug key, resistance coils, and connecting wires.

Currents in Multiple Circuits

When the current can proceed along two or more paths, between any two points of a circuit, it divides, and part of it goes along each path. If R_1, R_2, &c., are the resistances of each of the paths, $1/R_1$, $1/R_2$, &c., are called the "conductances" of the paths, and the current along each path is proportional to the conductance of that path. The conductance of the paths together is equal to the sum of the conductances of all the paths, and the ratio of the current through one path to the total current is equal to the conductance of that path, divided by the sum of the conductances of all the paths.

These statements may be proved as follows:—

Ohm's Law, applied to a length of wire through which a current flows, and in which there is no generator of electric current, states that the current is equal to the Electromotive Force or difference of potential between any two cross sections, divided by the resistance between the same sections. Thus, if two points, A and B, having a difference of potential V, are joined by a number

of wires having resistances R_1, R_2, etc., and if the currents through these wires are C_1, C_2, etc., respectively, we have the equations

$$C_1 = V/R_1,\ C_2 = V/R_2,\ C_3 = V/R_3, \text{etc.}$$

Calling the conductances K_1, K_2, etc., the equations become

$$C_1 = K_1V;\ C_2 = K_2V;\ C_3 = K_3V;\ \text{etc.}$$

Hence the currents through the separate wires are proportional to the conductances.

Adding up the equations, remembering that the whole current C is equal to the sum of the currents C_1, C_2, C_3, flowing through the separate wires, we find

$$C = (K_1 + K_2 + K_3 + \ldots\ldots)V.$$

Hence

$$C_1/C = K_1/(K_1 + K_2 + K_3 \ldots\ldots).$$

If we had a single wire, such that with the same difference of potential V between its ends, the same current C would flow through it, the conductance K would be given by the condition

$$C = KV.$$

Hence

$$K = K_1 + K_2 + K_3 + \ldots\ldots$$

and

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots$$

where R is the resistance of the single wire.

Example.—A Leclanché cell, having an EMF of 1·47 volts (a volt being the unit of Electromotive Force or difference of potential) and an internal resistance of 2 ohms, has its terminals connected by each of two wires of resistance 1 and 2 ohms. Find the total current flowing from the cell and the current in each wire.

The Resistance Bridge

Let a cell C be connected to the system of wires AR_1, BR_2 so that between the points A and B the current proceeds along the paths AR_1B, AR_2B (Fig. 74).

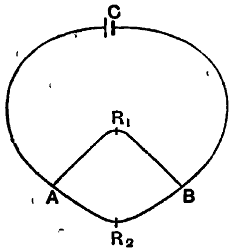


FIG. 74.

Then a certain difference of potential exists between the points A and B, and if we suppose A to be at a higher potential than B, the potential will vary along each of the wires AR_1B, AR_2B from its value at A to its value at B. Let R_1 be a point on the wire AR_1B. Then there must be some point on the wire AR_2B, which is at the same potential as R_1, and if the terminals of a galvanometer were connected to R_1 and to the point on the other wire at the same potential, there would be no current through the galvanometer. Let R_2 be the point at the same potential as R_1, and let us find the relation between the resistances of AR_1 and BR_1, AR_2 and BR_2 in order that this may be the case.

By Ohm's law, we know that the EMF or difference of potential, between any two points in a conductor carrying a current, is proportional to the resistance of the conductor between the two points.

Hence we have

$$\frac{\text{difference of potential between A and } R_1}{\text{do.} \qquad \text{A and B}} = \frac{\text{resistance of } AR_1}{\text{do.} \qquad AR_1B}.$$

Similarly

$$\frac{\text{difference of potential between A and } R_2}{\text{do.} \qquad \text{A and B}} = \frac{\text{resistance of } AR_2}{\text{do.} \qquad AR_2B}.$$

Now, if R_1 and R_2 are at the same potential the fractions on the left sides of these equations are equal. Hence, equating the right members, we have

$$\frac{\text{resistance of } AR_1}{\text{do.} \qquad AR_1B} = \frac{\text{resistance of } AR_2}{\text{do.} \qquad AR_2B},$$

from which it follows that

$$\frac{\text{resistance of } AR_1}{\text{do.} \qquad R_1B} = \frac{\text{resistance of } AR_2}{\text{do.} \qquad R_2B}.$$

Hence, if we know the ratio of the resistances of AR_2 and R_2B, and find that a galvanometer connected to R_1 and R_2 gives no deflection, we know also the ratio of the resistances of AR_1 and R_1B.

The Astatic Galvanometer

In the tangent galvanometer, it was seen that the force due to the current passing through the coil and tending to deflect the needle from the magnetic meridian, was opposed by the force on the needle due to the magnetism of the earth. If by any means we could diminish the force on the needle due to the earth, then with the same current flowing round the galvanometer coils, we should get a greater deflection, that is our galvanometer would be more sensitive. In the astatic galvanometer this is secured by attaching the needle within the coil rigidly to another needle nearly like it, outside the coil, but with its poles turned the

opposite way. The forces on the needles due to the earth act in opposite directions, and it is only the difference between these forces which now opposes the deflection of the needle. On the other hand, by the fundamental laws given in Sect. XXXVII., the forces due to the current tend to produce rotations in the same direction. The galvanometer is thus rendered much more delicate, and the needle moves when a very small current passes through the coils.

Exercise

The astatic galvanometer provided (Fig. 75) is to be placed on the table with the plane of the windings parallel

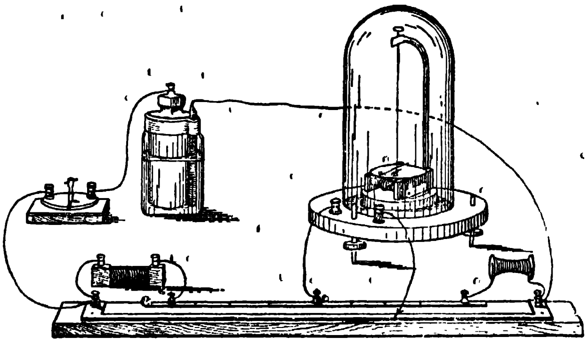


FIG. 75.

to the needles when the latter hang freely, and rotated in a horizontal plane and adjusted by means of the levelling screws, till the upper needle moves freely over the small scale between the stops and comes to rest with one end over the middle division of the scale. When this is the case the coils are in the magnetic meridian.

Connect a Leclanché cell through a plug key K to the screws A, B of the Resistance bridge (Figs. 75 and 76). The plug must be taken out when observations are not

being made. In the gap P place the 1-ohm resistance coil provided. In Q place a piece of No. 34 copper wire 1 metre long. Connect one terminal of the galvanometer to G, and attach to the other terminal a wire C, the other end of which is connected to the terminal of the sliding contact of the bridge wire, or, if there is no slider, is inserted in a cork in such a way that by holding the cork in the hand, the end of the wire can be brought into contact with the wire of the bridge.

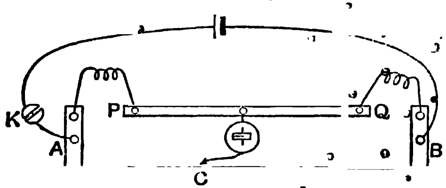


FIG. 76.

Insert the plug K, and make contact for an instant between C and the middle of the bridge wire. The needle of the galvanometer will probably be deflected. Alter the point of contact of C with the wire till, on making or breaking contact, no effect is observed on the needle. Take a reading of the position of C on the scale under the wire. Repeat this determination three times, and take the mean x of the readings. If the bridge wire is uniform and 100 divisions long, the resistances of the two parts, AC and CB, are in the ratio x to $100-x$, and if P is the resistance of the coil, Q that of the wire, we have

$$\frac{Q}{P} = \frac{100-x}{x}$$

or

$$Q = P\left(\frac{100}{x} - 1\right).$$

If P is known this gives the value of Q.

Take a piece of No. 29 platinoid wire, 30 cms. long, and place it in one of the gaps of the bridge, and determine its resistance.

Similarly determine the resistances of 15 cms. of the same wire, and of 30 cms. of No. 25 wire, and 40 cms. of No. 34 iron wire.

The resistance of a wire of length l, and diameter d, is proportional to l, and inversely proportional to d^2, and depends, in addition, on the material of the wire.

The quantity $\frac{\text{Resistance} \times \text{cross section}}{\text{length}}$ is called the resistivity or specific resistance of the material of the wire. It represents the resistance of a cube of the material of 1 cm. edge.

Determine by the wire gauge the diameters of the wires used.

From the observed resistances and the dimensions (see table, p. 239) of the wires calculate the specific resistances, and tabulate as follows:—

Bridge No. 2 Galvanometer No. 4. Coil No. 5
Resistance of coil No. 5 = 1·02 ohms = P.

Wire.	Reading.	Resistance wire / do. coil	Resistance of wire	cms. diameter of wire.	Cross section of wire	Resist × section / length.
100 cms. copper No. 34.	74·3	·346	·353	·023	·00043	·000,001,4
30 cms. platinoid No. 29.						
15 cms. do.						
30 cms. platinoid No. 25.						
40 cms. iron . . No. 34.						

It will be evident from the observations, that the specific resistance of platinoid is considerably greater than that of copper.

SECTION XL

Comparison of Electromotive Forces

Apparatus required. — Astatic galvanometer, wire stretched on scale, a storage cell, other cells, a key, and connecting wires.

We have seen that when a current passes along a wire, any two points of the wire differ in potential, by an amount which is equal to the product of the current passing through the wire, into the resistance of the wire between the two points. Suppose we have a cell, the EMF of which is equal to this difference of potential, and we connect the negative terminal to the point of the wire which has the lower potential. The terminal of the cell and the connecting wire attached to it, are then at the same potential as the point of the original wire with which the connecting wire is in contact. If we connect the positive terminal of the cell with the other point by means of a second wire, no current will flow along this wire through the cell, since the two ends of the wire are at the same potential, but if the second connecting wire be made to touch any other point of the original wire, a current will flow through the cell, the direction of the current depending on the side of the former point at which contact is made. By placing a galvanometer in series with the cell, the position of second point of contact in the wire for which no current through the cell is produced can therefore be found. If then the resistance of the original wire between the two points of contact, and the current through the wire are known, the EMF of the cell which is the product of the two, may be found. If the cell be replaced

by another having a different EMF, the distance between the points of contact of the connecting wires for no current through the cell will be different, and if the current through the wire is the same in the two experiments, the ratio of the lengths of wire between the points of contact in each experiment, will be the ratio of the electromotive forces of the cells.

Exercise

A thin platinoid wire AB is stretched along a graduated scale, and the ends of the wire are connected through a plug key K to the terminals of a storage cell C (Fig. 77). Care must be taken to ascertain that the stretched wire is of a sufficiently high resistance to prevent the storage cell from sending too large a current through it, and running down rapidly. The plug K must be inserted only when an observation is being made.

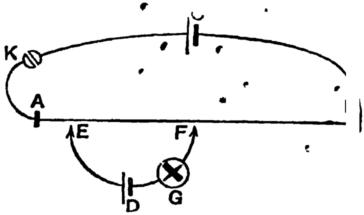


Fig. 77.

The Leclanché cell D, which is to be tested, is placed in series with an astatic galvanometer G, in such a way that the negative pole of the cell (the zinc) can be connected by the wire E and a sliding contact to that end of the stretched wire which is connected to the negative pole of the storage cell (the pole painted black) or to any other point on the wire. The wire F, attached to the galvanometer, can also be connected to the stretched wire by means of a second sliding contact.

Bring E into contact with the end of the stretched wire, and find a point along the wire with which F may be connected, without the galvanometer needle being deflected. Take the reading of the point. Now place E at the division 10, and repeat the determination. Similarly with E at 20, 30, etc., till E reaches the other end of the scale.

Now substitute for the Leclanché a Daniell cell, and go through the same series of observations. Replace the Leclanché, and repeat the original observations.

The object of this repetition is to eliminate errors due to a possible decrease of the current from the storage cell during the experiment. The point E is moved along the stretched wire, so as to get rid of errors due to the want of uniformity of the wire, since we are about to assume that the resistance of the wire between any two points is proportional to the distance between the points, and this is only true if the wire is uniform.

Tabulate the observations as follows:—

Wire No. 2. Storage cell No. 4. Galvanometer No. 1.

Cell.	Reading E	Reading F.	Difference	Mean.
Leclanché No. 3.	0 cms.	70.2 cms.	70·2 cms.	
	10	79·8	69·8	
	20	90·1	70·1	
	30	99·9	69·9	70·0
Daniell No. 5.	0	55·0	55·0	
	10	64·8	54·8	
	20	75·1	55·1	
	30	83·9	53·9	
	40	92·8	52·8	54·3
Leclanché No. 3.	0	71·0	71·0	
	10	80·8	70·8	
	20	91·0	71·0	
	30	100·7	70·7	70·9

$$\frac{\text{EMF Leclanché}}{\text{EMF Daniell}} \cdot \frac{70{\cdot}4}{54{\cdot}3} = 1{\cdot}29.$$

Take observations for a simple cell of zinc and copper in dilute sulphuric acid, and then again for the Leclanché cell, giving the whole of the observations and results in tabular form as above. Note that the EMF of the simple cell varies rapidly.

SECTION XLI

The Passage of Currents through Electrolytes

Apparatus required.—A water voltameter, storage cells, and a tangent galvanometer.

When an electric current is sent through certain liquids between electrodes on which they have no chemical action the liquids are decomposed into two constituents, one of which appears at each electrode. Such liquids are called electrolytes. The amount of decomposition produced is proportional to the amount of electricity sent through the liquid, a relation which is known as Faraday's first law of electrolysis. If a current C flows for t seconds, the amount of electricity which has passed in the time is Ct, and if m grams of the electrolyte are decomposed we have

$$m = KCt$$

when K is a constant depending on the nature of the liquid.

Exercise

The electrolyte to be decomposed in the present exercise is water, to which a little acid has been added to render it electrolytic. The water is placed in an N-shaped glass tube provided with two platinum electrodes through which the current enters and leaves the liquid. The gas which ascends from one electrode collects in the closed limb of the tube, and its volume is indicated by the graduations on the tube or on the stand. Such an arrangement is called a water voltameter (Fig. 78).

To verify that the amount of gas liberated is proportional to the product of the strength of the current, into the time for which it has passed, fill the closed tube of the

voltameter and the middle tube with dilute acid up to top of the middle tube by tilting it, and connect the voltameter, two storage cells, a plug key as a convenient make and break, a tangent galvanometer, and a 2-ohm coil in series. In order that the hydrogen may collect in the closed tube, the platinum foil in that tube should be connected to the negative, *i.e. black* terminal of the storage cell. Take the reading of the zero of the galvanometer. Insert the key, and observe whether the deflection on the galvanometer is readable, and the rate

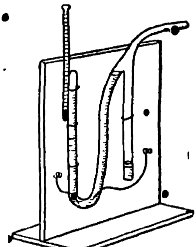


FIG. 78.

of production of gas such that one centimetre of the tube is filled in one or two minutes. Increase or decrease the resistance in circuit till this is the case.

Take the time at which the gas reaches the 1 cm. mark on the glass tube, read the galvanometer when the gas reaches 1·5 cm., observe the time at the second cm., read the galvanometer, take the time at the third cm., read the galvanometer, and finally take the time at the fourth cm.

Find the tangents of the angles of deflection on the galvanometer, multiply each by the time which the current has taken to generate the c.c. of gas, and show that the three products are approximately equal.[1]

Tilt the tube so as to refill the closed limb with the

[1] The correction due to the difference of level of the liquid in the two tubes may in the above experiments be neglected.

dilute acid. Alter the resistance in the circuit so as to get a current of different amount and repeat the observations. Show that the products of the tangents of the angles of deflection into the times, in the second series of experiments, are identical with those in the first.

Tabulate your observations as follows :—

Tube 3. Galvanometer 5.
Hydrogen collected.

c.c.	Time.	Interval Seconds.	Deflection.	Tangent.	Time × Tangents.
	H. M. S.				
1·0	11 15 5				
		72	27·5°	·52	37·4
2·0	16 17				
		73	27·5	·52	38·0
3·0	17 30				
		70	28·0	·53	37·1
4·0	18 40				

Temperature of gas 18° C. Barometer 75·2 cms.
Similarly for the other observations.

If the nature of the gas collected in the closed tube is known, the above experiment may be used to determine the constant of the tangent galvanometer used, *i.e.* to determine the multiplier which converts the tangents of the angles of deflection into amperes.

Let, for example, the gas in the above experiment be hydrogen. Then since 1 ampere flowing for 1 second liberates ·118 c.c. of hydrogen at 76 cms. and 18° C., and the volume generated in a given time has been observed, the current which passed can be found, and by comparison with the tangents of the angles of deflection, the current which would give a deflection of 45° can be calculated.

Calculate the constant of the galvanometer, tabulating your work as follows :—

Mean of times × tangents = 37·5.
Volumes at 75·2 cms. pressure = 1· c.c.
Volumes reduced to 76 cms. pressure = 75/76 c.c.

$$\therefore \text{ Constant of galvanometer} = \frac{75}{76}\Big/(37{\cdot}5 \times {\cdot}118) = \underline{{\cdot}223.}$$

PART VIII

ELECTRIC CHARGES

SECTION XLII

Electrification

Apparatus required.—Glass and ebonite rods, rubbers, electroscope, electrophorus, insulated spheres, and cans.

Before commencing this exercise, students should make themselves familiar with the elementary laws of electrical attraction and repulsion, explained in text books.

The experiments are to be performed in the order here indicated, the distribution of the electric charges at the different stages shown by diagrams in the note-books, and reasons given for the effects which are observed.

Before commencing, the glass and ebonite rods and the rubbers should be dried.

I. Electrification by Friction

a. Rub a glass rod with silk, and hold it over some small pieces of paper on the bench. They are attracted, showing that the rod is electrified.

b. Rub an ebonite or a sealing wax rod with flannel, and show that it is electrified.

c. Suspend an electrified glass rod horizontally by means of the silk thread provided, and show that another electri-

fied glass rod repels it. Similarly two electrified ebonite rods repel each other.

d. Show that an electrified ebonite rod attracts an electrified glass rod more than an unelectrified ebonite rod does.

II. Electroscope

The electroscope provided (Fig. 79) is surrounded by a metal case, in order that the leaves may be affected as little as possible by electrified bodies in the neighbourhood.

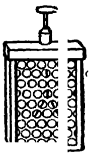


Fig.

a. Charge the instrument by bringing an electrified glass rod into contact with the circular disc or plate at the top.

b. Bring an electrified glass rod near the plate without touching it, then remove it again. Record what takes place.

c. Bring a strongly electrified ebonite rod towards the electroscope. If the charge given to the electroscope was not too large, the leaves will collapse and again diverge. Remove the ebonite, and discharge the electroscope by touching the plate with the hand.

d. Bring an electrified glass rod near but not touching the plate of the discharged electroscope. Touch the plate with the hand for an instant and then remove the glass rod. The leaves diverge. Explain the reason for this. The electroscope is said to have been charged by "Induction."

e. Bring an electrified glass rod towards the plate of the electroscope; the leaves will collapse and again diverge. Bring a charged ebonite rod towards the plate, and the leaves diverge further.

Compare these observations with those of *b* and *c*, and give the reason for the difference.

III. Electrophorus

a. Charge the electroscope by touching it with an electrified glass rod.

b. Excite the ebonite disc of the electrophorus (Fig. 80) by rubbing it with flannel. Place the plate on the ebonite, touch it for an instant with the finger, and then raise it by means of the insulated stem.

FIG. 80.

c. Bring the plate of the electrophorus near to that of the electroscope. The leaves of the electroscope diverge further. If the ebonite disc be brought near the electroscope plate, the leaves collapse and again diverge. Give the reason for these results.

IV. "Induction"

a. Place two uncharged insulated brass knobs in contact. Bring the charged plate of the electrophorus near one of them. Without touching the knobs separate them while the electrified plate is near, and show by bringing them in succession near the plate of the charged electroscope, that they are charged differently.

b. Bring the knobs into contact, and show that neither has now any effect on the electroscope.

c. Discharge the electroscope, and place on a block of paraffin, the can with which you are provided, and by means of a thin wire connect it to the plate of the instrument.

If the plate of the electroscope is firm enough the can may be placed directly on it.

d. Charge the electrophorus, and bring the plate gradually down into the can. The leaves of the electroscope diverge and the divergence increases till the plate is well within the can. then remains the same even if the

plate be made to touch the inside of the can. If the plate be removed and the ebonite disc of the electrophorus be now brought similarly into the can by means of the silk threads attached to it, the leaves diverge as before.

e. Charge the electrophorus, but leave the plate in contact with the ebonite, and bring the two together into the insulated can. No effect is produced on the electroscope, showing that although both the ebonite and the plate are charged with electricity, the external effects of the charges neutralise each other.

On account of this property of neutralising each other, which the different kinds of electricity possess, one is called "positive" and the other "negative"; glass, when rubbed with silk, is said to be positively, ebonite rubbed with flannel to be negatively, electrified.

When electricity is generated by friction, equal quantities of positive and negative electricities are produced.

This can be proved by connecting a can, containing a smaller insulated can in which an ebonite rod can rotate against cat skin, to the plate of the electroscope.

If the ebonite is rotated, electricity will be produced by friction, but so long as the ebonite remains in the inner can, the leaves of the electroscope remain in contact. They diverge if the rod with its negative charge is withdrawn.

SECTION XLIII

Potential and Capacity

Apparatus required.—Electroscope, insulated metal plates and glass plate.

Before commencing the experiments, students should read the account of Potential and of Capacity given in some text-book.

I. Potential

The divergence of the leaves of the electroscope depends on the difference of potential between the leaves and the surrounding case.

Place the electroscope on an insulating stand (a cake of paraffin), connect the plate and case, and charge by bringing the charged plate of the electrophorus several times into contact with that of the electroscope. Although a large charge is given to the electroscope, no divergence of the leaves occurs.

Disconnect the plate and case, touching each to discharge them completely.

Give a succession of charges to the case, and observe that the leaves diverge. Stop the charging when the leaves are slightly diverged, touch the plate of the electroscope, and observe that the leaves diverge further, although the hand is in contact with the plate of the electroscope. Explain the cause of this by a diagram.

Remove the hand, and discharge the case by touching it. The leaves collapse partially. Discharge them more completely by touching the plate.

Place an inverted metal can on the top of the case of the electroscope, so that the plate of the instrument is almost entirely surrounded by metal. Repeat the above experiment, and notice that a strong charge can now be given to the case without the leaves diverging. Explain this.

Touch the case and can, so as to discharge them. Remove the can, and give the leaves a small positive charge, by touching the plate with the plate of the electrophorus. If the charge is too great, allow some of it to leak away by touching the plate with a strip of paper. Show that on bringing the recharged plate of the electrophorus near the plate of the electroscope, the divergence of the leaves increases. Charge the case by the electrophorus, and notice that as the charging is continued, the divergence of the leaves diminishes to zero and then increases. On now bringing the plate of the electrophorus near that of the electroscope, the divergence of the leaves diminishes. Explain this by means of diagrams.

Discharge the case, and again charge the electroscope, so that the leaves diverge a little. Touch the case, and notice that the divergence is increased. State the reason for this.

II. Lines of Force

Connect the plate of the electroscope by means of a thin wire to one of the smaller vertical insulated metal plates (Fig. 81).

Take a piece of thin wire, about 6 cms. long, and attach to one end by means of a little gum, a single cotton fibre about 2 cms. long, obtained by pulling to shreds a piece of cotton thread. Charge the metal plate, and hold

the wire so that the cotton fibre is nearly in contact with the plate. Notice that the fibre sets itself so as to be perpendicular to the surface of the plate. Move the fibre round the plate, and along the wire connecting the plate to the electroscope, without touching them. Note the direction which the fibre takes at the edges of the plate and along the wire. Owing to the presence of the charge

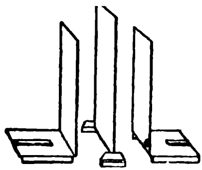


FIG. 81.

on the plate, an opposite charge is induced on the end of the fibre, the charge of like kind to that on the plate being conducted though the fibre and wire to earth. The charge at the end of the fibre is attracted by the plate, and the short length of fibre near the end sets itself along the lines of force.

III. Variation of the Potential round a Charged Conductor

Remove the electroscope from the insulating stand and place it on the bench. The case will now be connected through the bench to earth, and its potential will therefore be zero. Hence the divergence of the leaves will indicate the potential of the body with which they are connected. If that potential is positive, then a positive charge brought near the plate of the electroscope increases the divergence, if it is negative the charge decreases the divergence of the leaves of the instrument.

The potential of an insulated positively charged conductor

not in the neighbourhood of other conductors is positive, and the potential at points in the air surrounding the conductor diminishes on all sides. To show this, charge the larger of the metal plates (Fig. 81) by means of the electrophorus. Connect one of the smaller plates to the electroscope, and move it, by means of the insulated handle, towards the charged plate. The leaves diverge, and the divergence increases as the plates approach. Show, as above, that the potential of the smaller plate is positive.

To charge the larger plate negatively by means of the electrophorus, place the plate of the electrophorus on the ebonite, and instead of touching it to take away the negative charge, connect it for an instant, by means of an insulated wire held by a sealing wax handle, to the large plate. The negative charge then goes to the metal plate. Raise the plate of the electrophorus, touch it, and repeat the above operations.

Prove, as above, that the potential of the metal plate is negative, and that the potential increases algebraically on all sides of the negatively charged metal plate.

IV. Capacity

Connect the large plate to the electroscope, remove the smaller plate to some distance, and touch it. Charge the larger plate and electroscope, the divergence of the leaves of which indicates the potential to which the two have been charged. Now bring the smaller plate, which is at zero potential, towards the larger plate; the divergence of the leaves of the electroscope diminishes, hence the potential of the large plate diminishes. We have seen above that the potential of the smaller plate increases. Thus the two plates mutually influence each other. Touch the smaller plate, and notice that the divergence again diminishes. If the plates are very near together, on touching the smaller, the divergence of the leaves can be diminished to a very small amount. To cause the original amount of divergence, it

is necessary, if the smaller plate be kept connected to earth, to give a great number of charges to the larger plate, or to remove the smaller plate from the larger. The quantity of electricity which it is necessary to give to a conductor, in order to raise its potential by unity above that of neighbouring earth connected conductors, is called the "Capacity" of the conductor in its particular surroundings; and we see from the above experiment that the capacity of the arrangement is increased by bringing the plates nearer together. On account of the large capacity of a plate close to another plate connected to earth, the system is called a "condenser."

If a negative charge is given to the smaller plate which was touched, its potential will be negative, and there will be some point between the plates which has the potential of the earth.

Discharge the electroscope and charge one of the smaller metal plates negatively, as described on the previous page, giving the other plate the positive charge. Place the two plates on opposite sides of the large plate, and about 4 cms. apart. Move one of them about parallel to the others till the divergence of the leaves is zero. The large plate ought, if the charges on the outer plates are equal, to be half way between them. If the charges are not equal, the position will be nearer the plate having the smaller charge.

V. Influence of the Dielectric between the Plates

Arrange the two smaller plates opposite each other with a plate of glass between them. Connect one by means of a wire to the electroscope. Charge it, and touch the other plate, so as to bring its potential to zero. Withdraw the glass plate suddenly, and notice that the divergence of the leaves of the electroscope increases. Show from this experiment that the dielectric constant of glass is greater than that of air.

APPENDICES

APPENDIX A

ADDITIONAL EXERCISES

SECTION XXI. A.—To determine the coefficient of expansion of a gas with temperature, the pressure remaining constant.

Apparatus required.—Graduated capillary tube containing air, closed by sulphuric acid index, water-bath, thermometer.

When the pressure to which a gas is subjected remains constant, the volume v_t the gas occupies at any temperature t°C is given in terms of the volume v_o at 0°C by the equation $v_t = v_0(1 + at)$, where a is a constant known as the "coefficient of expansion of the gas at constant pressure."

It is the object of the present exercise to determine the quantity a.

The capillary tube supplied is closed at the end from which the graduations start. It contains air, and is closed at the other end by a column of strong sulphuric acid, which extends a few millimetres up the wider tube attached to the capillary tube. The acid serves as an index and keeps the air dry.

Place the tube horizontally in the water-bath supplied, and fill the bath with tap water, the end of the wider tube being kept well above the surface of the water. Place a centigrade thermometer in the water and stir well. When the indication of the thermometer is constant, read the temperature, and the position of the sulphuric acid index in the capillary tube. If ice is available, add sufficient to the water to lower its temperature to 10°C, and repeat the observations of temperature and volume. Then pour out the water and surround the tube with ice. When the temperature has fallen to 0°C, read it and the volume. Place a Bunsen burner under the

water-bath and raise the temperature to 10°C, remove burner and repeat the observations, after stirring the water well. Continue the process, taking readings of volume and temperature at about 20°, 30°, 40°, 50°, 60°, and 70°C, before each reading stirring the water well and keeping its temperature constant for a few minutes by turning down the burner. Now add cold water to the bath, so as to reduce its temperature by steps of about 10°C to that of the air, reading temperature and volume at each step.

Record the observations graphically, taking distances to the right in the squared paper of your note-books to indicate temperature, and distances upwards to indicate volumes. Draw through the points so obtained the straight line which best represents the observations.

By means of this straight line, read the volumes v_{50} at 50°C and v_0 at 0°C. Divide the former by the latter, subtract 1 from the quotient and divide the difference by 50. The quotient is the coefficient of expansion of the gas at constant pressure α; the expansion of the glass of the tube and the slight change of pressure due to the motion of the sulphuric acid up the wide tube being negligable.

Record the calculation as follows :—

Tube B.

$v_0 = 10\cdot20$ scale divisions of tube

$v_{50} = 12\cdot05$,, ,, ,,

$$\frac{v_{50}}{v_0} = 1\cdot181 \qquad \frac{v_{50}}{v_0} - 1 = \cdot181,$$

$$\therefore \alpha = \frac{\frac{v_{50}}{v_0} - 1}{50} = \cdot00361.$$

At the conclusion of the exercise pour the water out of the water-bath so as to leave the gas tube dry.

SECTION XXI.B.—To Determine the Dew Point and the Relative Humidity or Fractional Saturation of the Air.

Apparatus Required.—Daniell hygrometer, ether.

The air generally contains water vapour, and the temperature to which the air has to be cooled in order that the vapour may condense on bodies with which it comes into contact is known as the Dew Point. It is the temperature at which the amount of vapour present at the time would be sufficient to saturate the air.

The Daniell hygrometer supplied consists of an inverted U-shaped tube, with the legs unequal in length and each provided with a bulb. The lower bulb is generally silvered or gilded, and within it the bulb of a small thermometer is placed. The upper bulb is covered with muslin. The whole tube has been exhausted so that it contains a volatile liquid and its vapour only. At the commencement of the exercise the hygrometer should be tilted so that the whole of the liquid is in the lower bulb. Allow the instrument to stand in the laboratory for ten minutes before using. Then read the thermometer in the lower bulb and that outside, generally on the wooden stand of the instrument. If the two readings do not agree, we shall take the thermometer within the tube as correct, and apply a correction to the readings of the outer one so as to bring its readings into agreement with the inner one.

Now place the hygrometer outside at a point in the shade where there is a free circulation of air. Pour a little ether over the upper bulb so as to saturate the muslin, and keep it saturated during the experiment. Notice that the thermometer in the bulb shows a fall of temperature. Shake the instrument occasionally so as to stir the liquid in the bulb, and watch the outer surface of the bulb for the first trace of moisture deposited on it. This will be most easily detected by looking at the image in an object seen by reflection is the silvered or gilded surface. When the first trace of moisture appears, the image will become dim. Read immediately the temperatures on the inside and outside thermometers.

Allow the ether on the muslin to evaporate completely. The temperature of the hygrometer will then rise slowly. Shake the instrument at intervals so as to stir the liquid in the bulb, and watch the instant when the film of moisture on the bulb disappears. Read immediately both thermometers.

Take the means of the readings of each thermometer. That for the inner thermometer is the Dew Point. Correct those of the outer one if necessary, and find from tables of the saturation pressure of water vapour at different temperatures, the pressures corresponding to the Dew Point found, and to the corrected temperature of the air.

Since the amount of vapour present at the latter temperature was sufficient to saturate the air at the observed Dew Point, the relative humidity or fractional saturation is equal to the quotient of the former of these pressures by the latter.

Record as follows :—

DANIELL HYGROMETER B.

Readings of thermometers after standing in room.

Inside 19·2°C.

Outside 18·8°C. ∴ Correction to outside readings = ·4°C.

Readings for Dew Point.

					Vapour pressure.
Inside . .	12·4°	12·6°	mean	12·5	1·08 cms. mercury
Outside . .	17·4	17·4	,,	17·4	
Correction .			,,	·4	
				17·8	1·44 ,,

$$\therefore \left.\begin{array}{l}\text{Relative Humidity or}\\ \text{Fractional Saturation}\end{array}\right\} = \frac{1 \cdot 08}{1 \cdot 44} = \cdot 75.$$

APPENDIX B

We propose in this appendix to give some information which we hope will prove useful to teachers.

Students, when entering a Physical Laboratory, require some instruction concerning their note-books and the proper treatment of apparatus. The following regulations, which are issued to the students of the University at the beginning of each session, may serve as a guide to the teacher.

REGULATIONS

I. Each student should provide himself with two note-books.

(1) A note-book in which to record observations as they are taken and to make rough calculations. This must be available for inspection when required. No observations are to be taken, or calculations made, on loose sheets of paper.

(2) A note-book in which to write out carefully the method of working and the results obtained. Alternate pages of this book should be divided into squares.[1]

II. He should also be provided with the following drawing instruments:—pen and pencil compass, spring bow dividers, boxwood rule, having diagonal and protractor scales, a six-inch steel rule divided into millimetres and inches.

III. Each student is responsible for the apparatus he is using, and all pieces of apparatus must at the end of the class be left in the same condition, and in the same place, in which they were found.

[1] For the sake of uniformity it is desirable that note-books of the same kind should be used by students.

IV. Each piece of apparatus is numbered, and students must enter in their note-book the number of each piece they use. Students failing to comply with this Regulation are liable to be made responsible for any damage which cannot be traced to its author.

V. Students are not allowed to leave an exercise and proceed to another, until their work has been found satisfactory and approved. A description of the approved exercise and results should then be written in the second or principal note-book.

VI. The principal note-book of each student will be examined from time to time, and the character of the records made in it will be taken into account in making out the class list at the end of the course.

The exercises described in this volume need not necessarily be performed in the order in which they are given, and by judicious arrangement, a considerable saving in apparatus may be effected. Thus it will be found possible, with the help of five or six sets of apparatus, to teach thirty students, by having five or six different experiments going during each lesson. The principal difficulty will be found at the beginning of the session, when all students start from the same level. If, however, they possess, as they should do upon entering on a Laboratory Course, some knowledge of Theoretical Mechanics, the exercises in Part II. may be taken in almost any order. After a few weeks, as some of the students will have had to repeat the exercises, and others will have gone ahead quickly, they will be separated sufficiently to allow the exercises to be taken in the order given.

In order that this separation of students should not become too great, some of the more difficult exercises may be omitted by those who have fallen behind. It is better for a weak student to be made to repeat the same exercise until he has mastered it, than for him to be allowed to perform experiments he does not understand.

To enable the teacher to arrange the work of the class satisfactorily, he should keep a clear record of what each student has done. The following method has proved satisfactory.

The names of the students are entered in a column at the left hand of a sheet of paper ruled in columns, each of which refers to a meeting of the class. Each student thus has a square assigned to him for each date of meeting, and in this square the number of

the section of the book at which he happens to be working, is entered. When a student has performed an exercise, and his result has been considered satisfactory, the sign – is entered in the square above the number of the section. When the experiments and the results have been written out neatly in the note-book, and have been approved and initialled by the teacher, a vertical stroke is added, and the resulting sign + above the number of the section indicates that the exercise is complete. The following table will show the appearance of such a "Work Sheet." Reference to it will show at once that student A has completed sections 6 and 7, that he has performed the exercises of sections 5 and 8 satisfactorily, but has not yet shown the results written out in proper form, and that he was absent on October 30.

Name.	October				
	2	9	16	23	30
A	+ 6	+ 7	– 5	– 8	a
B	+ 7	+ + 8 5	+ + 6 9	+ 10	11
C	+ 5	+ 6	+ 7 8	– 8	9
D	5	+ 5	6	– 6	7
E	+ 8	+ 9	+ + 7 5	– 6	10
F	+ 6	+ 7	a	– 5	8

SECTION I

Teachers will find it useful to get students to practice sub-division by eye estimate, and the following method may be adopted. Rule on paper a number of lines beginning and ending sharply, and varying in length from about 5 mms. to about 3 cms. Then place a sharp mark at random on each line, and let the students estimate the distance of the mark from one end of the line, in tenths of the whole length. The estimate may then be verified by direct measurement.

SECTION V.—**The Vernier**

Both 'A' and 'B' scales and verniers are marked on the same piece of apparatus, the distance apart of the divisions of each scale being 1 cm. The 'A' and 'B' observations therefore check each other.

As an additional exercise the volume of the block of wood may be calculated.

SECTION VI.—**The Spherometer and Screw Gauge**

A spherometer with its legs about 4 cms. apart, and a screw gauge capable of measuring up to 1·5 or 2 cms., are sufficient.

SECTION VII.—**The Law of Moments**

The prices given on page 237 include separate supports for the apparatus required in Sections VII., VIII., and XIII. The fittings of the laboratory benches may enable these separate stands to be dispensed with; but there is an advantage in having each instrument portable and complete in itself. The discs are 25 cms. in diameter. The pegs should project just sufficiently from the discs to prevent the strings touching the discs.

SECTION VIII.—**The Pendulum**

Bobs of different materials may be taken, in order to prove that the value of *g* obtained is independent of the material of the bob.

If there is no clock with a seconds-hand in the class-room, watches suitable for the experiment may be obtained at a cost of 2*s*. 6*d*. or 2*s*. 9*d*.

SECTION IX.—The Hydrometer

The hydrometer which has been found most suitable for the purpose, has a total weight of 55 grs. The hollow cylinder, which serves as a float, has a length of 9 cms., a diameter of 2·8 cms., and is made of thin sheet brass.

The glass jar has a height of 33 cms., a diameter of 6 or 7 cms., and is filled with the liquid to such a level that when the hydrometer sinks it rests on the bottom of the jar before the upper pan touches the surface of the liquid.

Students sometimes get into difficulties by not placing the weights symmetrically in the pan. The hydrometer will then lean over, and the friction against the sides of the jar will make the weighings unreliable.

SECTION X.—The Balance. I

The balance is of brass and has arms about 15 cms. long. It will carry about 200 grams, and is sensitive to half a centigram.

SECTION XI.—The Balance. II

A piece of wood soaked in melted paraffin is convenient for Exercise III.

SECTION XII.—The Barometer

This section has been added because many laboratories are provided with a good barometer, and it is useful to medical students and others to be able to read such an instrument. The section, may, however, be omitted where a suitable barometer is not available. A very simple instrument, which may easily be made in the laboratory, will be sufficient for the purposes of Section XV.

SECTION XIII.—Elasticity

Round vulcanised rubber chord about 5 mms. diameter are suitable. The common hexagonal iron weights are quite accurate enough for this exercise.

If note-books with pages ruled in squares are used, the curves may be drawn in the note-book. Otherwise, curve paper may be obtained from any large stationer.

SECTION XIV.—Boyle's Law

Care should be taken that the indiarubber tube is sufficiently strong. Otherwise, on increasing the pressure the indiarubber will expand, and the surface of the mercury in one or other of the tubes may sink out of sight, so that it cannot be read off. Canvas lined pump tubing is suitable.

SECTION XV.—Freezing and Boiling Points

Paper scale thermometers, which are rarely half a degree wrong, may be obtained at a cost of about 1*s.* from several firms.

SECTION XVI.—Comparison of Thermometers

The vessel for heating the water consists of a brass can, 8 cms. in diameter and 10 cms. deep, standing on three legs at a convenient height for heating with a Bunsen burner.

The calorimeter in the specific heat exercises is made of thin copper, is 5 cms. in diameter and 9 cms. deep, and weighs 50 grams. It is supported on three cork legs, in an outer brass vessel 8 cms. in diameter and 12 cms. deep.

SECTION XVII.—Specific Heat. I

An exercise may be introduced here to show the gradual cooling of the water in a calorimeter. For this purpose fill the calorimeter to two-thirds its height with water at about 50°, keep the water well stirred, and take minute readings of a thermometer placed in it.

SECTION XIX.—Method of Mixtures

The tubes A and K are of brass, A 2 cms. diameter, 16 cms. long, K 4 cms. diameter, 18 cms. long.

It is convenient to determine the specific heat of a good conductor, so that the body will quickly give up its heat to the water in the calorimeter. Metal borings or turnings are suitable, but we have found marble to answer best on the whole. Small pieces of quartz which are unsuitable for optical purposes, and therefore of no value, may easily be obtained and used for the purpose. It is an advantage occasionally to vary the substance used.

As another exercise, the specific heat of a liquid may be determined by heating a solid of known specific heat, and plunging it into the liquid.

SECTION XX.—Latent Heats

It is only with some hesitation that we introduce the exercise on the latent heat of steam, as a simple apparatus does not give very satisfactory results. The arrangement recommended has been adopted after carefully trying all the methods usually given, and finding them unsatisfactory. The condenser here introduced has the additional advantage that the latent heats of the vapours of other liquids, *e.g.* alcohol and benzene, may be found with the same apparatus. It is made of thin sheet brass and thin brass tubing.

SECTION XXI.—Melting and Boiling Points

Naphthalene has been chosen for the determination of the melting point, wax and paraffin, generally used for this purpose, not having definite melting points.

Alcohol is not suitable for the determination of the boiling point, as it absorbs moisture, and boils at temperatures which differ according to the amount of water present.

SECTION XXII.—Reflection

Strips of mirror glass 5 cms. long and 1 cm. broad answer the purpose. The sighting rod consists of two needles soldered into a brass wire about 14 cms. long.

SECTION XXIII.—Refraction

The cubes used in this exercise, as well as the prisms used in Section XXVIII., may be obtained from most dealers. The edges of the cubes are 4·5 cms. long, and parallel to one edge and 1 cm. from it a line is scratched with a diamond.

SECTION XXV.—Lenses and Mirrors. II.

The convex lens used is the ordinary 3 in. focus circular double convex lens, and the concave lens the ordinary 6 in. circular double concave lens of the spectacle makers. The mirrors are of diameter 2 and focal length 3 or 4 inches.

SECTION XXVI.—Lenses. III.

The lens is the 3 in. focus double convex lens, and the mirror of 3 or 4 in. focus.

SECTION XXVIII.—**Refractive Index of a Glass Prism**

The lenses of collimator and sighting board are the ordinary 5 in. focus circular double convex lenses. The prism is 4 cms. long and each edge of the base is 2·5 cms.

SECTIONS XXIX AND XXX.—**Vision**

These Sections are somewhat more advanced and may be omitted by many students. They are introduced chiefly for the benefit of medical students. The method, though simple, gives good results, especially for the near points. Teachers will probably be surprised to find how many students are short-sighted without knowing it, and in how many cases the left eye differs considerably from the right eye. When the teacher can spare sufficient time to check the numbers, it will be of great interest to keep a record of the results obtained in Exercise I. of Section XXX. The authors will be glad to receive the statistics, when a sufficient number of eyes have been examined.

SECTION XXXI.—**The Sonometer**

The equations given in this and other exercises should, of course be explained to the students in lectures. No. 28 S.W.G. pianoforte wire is used on the sonometers, which have their bridges about 50 cms. apart. Tuning forks may be obtained from musical instrument dealers, or from any manufacturer, *e.g.* Messrs. Valentine and Carr, Bowden Street, Sheffield.

SECTION XXXII.—**Resonance**

A paper tube sliding along a vertical glass tube, or one brass tube sliding within another, make good resonators.

SECTION XXXIV.—**Magnetic Forces**

The magnets used in mapping the lines of force are 10 cms. long, 1·2 broad, and ·5 thick. The compass is the small 'charm compass' of the shops.

For comparison with the lines of force due to a magnet, the lines due to two unlike poles should be drawn and suspended in the laboratory. See J. J. Thomson's *Electricity*, p. 61, or Maxwell's *Electricity*, vol. i. p. 170.

SECTION XXXV.—**Magnetic Survey**

The circular box of the magnetometer is 13 cms. in diameter, and the suspending fibre 18 cms. long.

A plan of the laboratory should be drawn, and the points at which the observations are to be made, indicated on it. If the direction and intensity of the force at one point be given, they can from the observations be found at the other points.

SECTION XXXVI.—Magnetic Fields

To support the vibrating magnet which is about 6 cm. long, form a double loop at the end of the fibre, by doubling it back on itself twice, and tying a knot at the point where the single and the quadrupled fibres join.

SECTION XXXVII.—Action of Currents

Ordinary insulated wire sold in coils as electric bell wire makes very good connecting wire. The compass is that used in Section XXXIV.

Connectors for joining the ends of two wires together may be obtained from any electrical screw and terminal maker.

SECTION XXXVIII.—Voltaic Cell and Tangent Galvanometer

Leclanché cells, size No. 3, with the carbon enclosed in a porous pot, and with an internal resistance of about 2 ohms, are most suitable. The tangent galvanometer has a coil of 8 cms. diameter, and a suspension 16 cms. long.

SECTION XXXIX.—Wheatstone's Bridge

The wire of the bridge is 50 cms. long.

The wires, the specific resistances of the materials of which are determined in this exercise, may be obtained as follows:—

Copper: Mr. Wm. Rickard, Ashbourne Road Mills, Derby.

Iron: Sold in bobbins as binding wire by ironmongers.

Platinoid: The London Electric Wire Co., Playhouse Yard, London.

SECTION XLI.—Electrolysis

This exercise may be omitted by the more elementary students, or where no suitable battery is available.

By connecting the storage cell so that the oxygen is collected in the closed tube, an additional exercise may be made.

Simple experiments on electromagnetic induction may be carried out with an astatic galvanometer, a few turns of wire, and a magnet.

The apparatus required for this volume can be supplied by Mr. George Cussons, Technical Works, Broughton, Manchester, and London. Some of it is, however, of such a simple nature that it could be made by any local worker in wood and metal.

If supplied in sets, the cost is approximately as follows :—

	£	s.	d.
Mechanics	5	0	0
This includes: Vernier models, sliding callipers, spherometer, small brass cylinder, glass scale and plate, pendulum and stand, hydrometer with jar and box of weights, balance, large brass cylinder, can and sinker; elasticity apparatus, with stand complete, and apparatus for Boyle's law.			
Heat	1	1	0
Apparatus for boiling points, freezing points, specific heat, latent heat, and melting and boiling points.			
Light	0	15	0
Drawing table on folding legs, mirror and support, sighting rod, glass scale, glass cube, lenses, slit, stops and screens, and apparatus required for Section XXX.			
Sound	0	12	6
Sonometer, two tuning forks, and resonance tube.			
Magnetism	1	6	6
Magnets, magnetoscope, steel and iron wire, magnetometer, and vibration can.			
Current Electricity	3	9	0
Compass needle and stand, tangent and astatic galvanometers, one 2 ohm and one 1 ohm resistance coil, Leclanché cell, Wheatstone bridge and resistances, potentiometer, small storage cell (9*s.*)			
Electrostatics	0	12	6
Electroscope, electrophorous, silk, flannel, and fur rubbers, ebonite, can, and condenser plates.			
Total . . .	£12	16	6

n	n^2	$\sqrt{n}$	$\frac{1}{n}$
1	1	1·00	1·00
2	4	1·41	·50
3	9	1·73	·333
4	16	2·00	·250
5	25	2·24	·200
6	36	2·45	·167
7	49	2·65	·143
8	64	2·83	·125
9	81	3·00	·111
10	100	3·16	·100
11	121	3·32	·0909
12	144	3·46	·0833
13	169	3·61	·0769
14	196	3·74	·0714
15	225	3·87	·0667
16	256	4·00	·0625
17	289	4·12	·0588
18	324	4·24	·0556
19	361	4·36	·0526
20	400	4·47	·0500
21	441	4·58	·0476
22	484	4·69	·0455
23	529	4·80	·0435
24	576	4·90	·0417
25	625	5·00	·0400
26	676	5·10	·0385
27	729	5·20	·0370
28	784	5·29	·0357
29	841	5·39	·0345
30	900	5·48	·0333
31	961	5·57	·0323
32	1024	5·66	·0313
33	1089	5·74	·0303
34	1156	5·83	·0294
35	1225	5·92	·0286
36	1296	6·00	·0278
37	1369	6·08	·0270
38	1444	6·16	·0263
39	1521	6·24	·0256
40	1600	6·32	·0250
41	1681	6·40	·0244
42	1764	6·48	·0238
43	1849	6·56	·0233
44	1936	6·63	·0227
45	2025	6·71	·0222
46	2116	6·78	·0217
47	2209	6·86	·0213
48	2304	6·93	·0208
49	2401	7·00	·0204
50	2500	7·07	·0200

n	n^2	$\sqrt{n}$	$\frac{1}{n}$
51	2601	7·14	·01961
52	2704	7·21	·01923
53	2809	7·28	·01887
54	2916	7·35	·01852
55	3025	7·42	·01818
56	3136	7·48	·01786
57	3249	7·55	·01754
58	3364	7·62	·01724
59	3481	7·68	·01695
60	3600	7·75	·01667
61	3721	7·81	·01639
62	3844	7·87	·01613
63	3969	7·94	·01587
64	4096	8·00	·01563
65	4225	8·06	·01538
66	4356	8·12	·01515
67	4489	8·19	·01493
68	4624	8·25	·01471
69	4761	8·31	·01449
70	4900	8·37	·01429
71	5041	8·43	·01408
72	5184	8·49	·01389
73	5329	8·54	·01370
74	5476	8·60	·01351
75	5625	8·66	·01333
76	5776	8·72	·01316
77	5929	8·77	·01299
78	6084	8·83	·01282
79	6241	8·89	·01266
80	6400	8·94	·01250
81	6561	9·00	·01235
82	6724	9·06	·01220
83	6889	9·11	·01205
84	7056	9·17	·01190
85	7225	9·22	·01176
86	7396	9·27	·01163
87	7569	9·33	·01149
88	7744	9·38	·01136
89	7921	9·43	·01124
90	8100	9·49	·01111
91	8281	9·54	·01099
92	8464	9·59	·01087
93	8649	9·64	·01075
94	8836	9·70	·01064
95	9025	9·75	·01053
96	9216	9·80	·01042
97	9409	9·85	·01031
98	9604	9·90	·01020
99	9801	9·95	·01010
100	10000	10·00	·01000

$\pi = 3{\cdot}1416.$ $\pi^2 = 9{\cdot}870.$

Angle.	Sine.	Tangent.	Angle.	Sine.	Tangent.
1°	0·017	·017	46°	0·719	1·036
2	·035	·035	47	·731	1·072
3	·052	·052	48	·743	1·111
4	·070	·070	49	·755	1·150
5	·087	·087	50	·766	1·192
6	·105	·105	51	·777	1·235
7	·122	·123	52	·788	1·280
8	·139	·141	53	·799	1·327
9	·156	·158	54	·809	1·376
10	·174	·176	55	·819	1·428
11	·191	·194	56	·829	1·483
12	·208	·213	57	·839	1·540
13	·225	·231	58	·848	1·600
14	·242	·249	59	·857	1·664
15	·259	·268	60	·866	1·732
16	·276	·287	61	·875	1·804
17	·292	·306	62	·883	1·881
18	·309	·325	63	·891	1·963
19	·326	·344	64	·899	2·050
20	·342	·364	65	·906	2·145
21	·358	·384	66	·914	2·246
22	·375	·404	67	·921	2·356
23	·391	·424	68	·927	2·475
24	·407	·445	69	·934	2·605
25	·423	·466	70	·940	2·747
26	·438	·488	71	·946	2 904
27	·454	·510	72	·951	3·078
28	·470	·532	73	·956	3·271
29	·485	·554	74	·961	3·487
30	·500	·577	75	·966	3·732
31	·515	·601	76	·970	4·011
32	·530	·625	77	·974	4·331
33	·545	·649	78	·978	4·705
34	·559	·675	79	·982	5·145
35	·574	·700	80	·985	5·671
36	·588	·727	81	·988	6·314
37	·602	·754	82	·990	7·115
38	·616	·781	83	·993	8·144
39	·629	·810	84	·995	9·514
40	·643	·839	85	·996	11·43
41	·656	·869	86	·998	14·3
42	·669	·900	87	·999	19·1
43	·682	·933	88	·999	28·6
44	·695	·966	89	1·000	57·3
45°	·707	1·000	90°	1·000	∞

INDEX

INDEX

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